On the Benefits of Empirical Mode Decomposition in Spatio-temporal EEG Analysis

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***Abstract* - Empirical mode decomposition (EMD) is an effective tool for the analysis of non-linear and non-stationary signals, which has been widely used in various application fields for noise reduction, feature extraction and classification. Due to its adaptive and data-driven nature, it has been introduced to electroencephalography (EEG) analysis to extract more accurate information during time-frequency and phase analysis, multi-channel signal processing, and brain connectivity network construction. In our paper we review the development of EMD and its variants, illustrating their benefits in spatiotemporal EEG analysis, and introduce some practical applications of EMD in EEG analysis. Finally, we discuss future opportunities in EEG analysis with the EMD method, and outline parallelization strategies to speed up EMD processing.**

Keywords – Empirical Mode Decomposition; Electroencephalogram; Time-frequency Analysis

I. Introduction

EEG is widely used as a diagnostic tool in epilepsy treatment, sleep studies, stroke recovery monitoring, BCI systems, and is one of the most important tools in studying human cognitive processes. Compared with medical imaging methods such as CT (computer tomography), PET (positron emission tomography), or MRI (magneto resonance imaging), one of the main advantages of EEG is its very high temporal resolution. With the increasing use of high-density EEG, the spatial resolution of EEG is also improving.

While EEG waveform analysis provides important information during, say, anesthesia or sleep diagnostics, on many occasions, the use of frequency and time-frequency features is mandatory. EEG spectrum is most frequently calculated from the Fourier transform coefficients, and time-frequency representation is normally based on short-time Fourier or continuous wavelet transforms. The fundamental problem in time-frequency analysis is the uncertainty principle of time and frequency; accurate localization of an event results in inaccurate frequency spectrum, and accurate spectrum estimation results in low temporal accuracy.

Empirical Mode Decomposition (EMD) is a new method developed relatively recently for the analysis of non-stationary and non-periodic signals [1]. Most natural signals, including physiological ones, belong to this category. The time-frequency analysis method based on EMD and the Hilbert transform not only does remove the constraints of uncertainty principle, but can also use the adaptability and data-driven nature of EMD to decompose the signal to narrowband components to describe the time-frequency characteristics of the non-stationary signals more accurately.

Empirical Mode Decomposition started to be used in EEG analysis more than a decade ago. The purpose of this paper is to give an overview of the EMD method and review the application of EMD in EEG signal processing and analysis. We begin with the fundamentals of EEG signals, followed by the development of the EMD method and its variants. We describe representative applications of EMD in pre-processing, artifact removal, cognitive processing and resting state analysis. Finally, we outline the parallel implementation strategies of EMD.

II. Fundamentals of EEG

Electroencephalography (EEG) signals contain a wealth of information about the physiological and pathological activities of the brain, hence is an important method to study the brain. However, the brain is also the most complex system known to humans, and the EEG is a kind of 3N (non-stationary, nonlinear, noisy) signal, where non-stationarity means the statistical characteristics of the signal are time-varying, and nonlinearity refers to multiplicative effects in the signal. Although there are quasi-stationary segments in the EEG signal that can last for about 0.25 seconds [2], the overall EEG signal is still non-stationary and nonlinear. The EEG signal detected on the scalp (macroscopic level) is actually a mixture of multiple sources active in different brain regions. These sources are represented by the time-varying global field potential of neural populations. Due to volume conduction, the source signals are mixed with varying intensity and phase when transmitted to the scalp, resulting in a non-stationarity and nonlinear EEG signal. Since the EEG signal is very weak (microvolt-level amplitude), it is easily contaminated by noise, including environmental noise and other electrophysiological artifacts, which will further increase the complexity and uncertainty of the EEG signals. In addition, the non-stationarity of EEG signals is also attributed to certain neurological diseases, such as epilepsy [3], Alzheimer's disease [4], and some physiological state transitions, such as the transition of sleep stages [5].

III. Introduction to Empirical Mode Decomposition

Empirical Mode Decomposition (EMD) is a signal decomposition algorithm proposed by Huang *et al.* [1] in

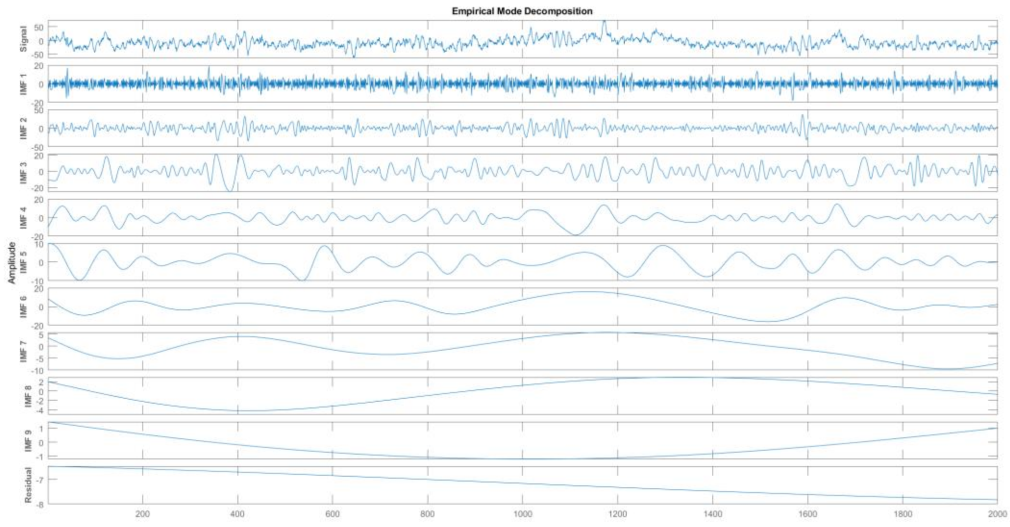

Figure 1. A sample EEG signal and its decomposed intrinsic mode functions. The last plot is the final residual signal.

1998 that decomposes the signals into a finite numbers of intrinsic mode functions (IMFs) [6].

EMD can decompose the signal according to the time scale features of the data itself, without any pre-set basis functions, which makes it fundamentally different from the Fourier or wavelet decomposition. In the Fourier decomposition, pre-defined sinusoid basis functions are used to separate the signal into components. In the wavelet decomposition, most commonly, a family of wavelet basis functions (e.g. Morlet) are used to describe different frequency ranges at different time scales.

In a non-stationary, non-periodic signal, there are multiple oscillation modes at any time instance, which means that the signal at each time point contains multiple instantaneous frequencies [7] so, the Fourier-based time-frequency analysis of non-stationary signals has only mathematical but no physical meaning. The EMD method can decompose the non-stationary signal into narrow band components. Fig. 1 illustrates the decomposition of an EEG signal into several narrow band intrinsic mode functions. These components then can be analyzed using the Hilbert transform for the exact instantaneous frequency and phase information.

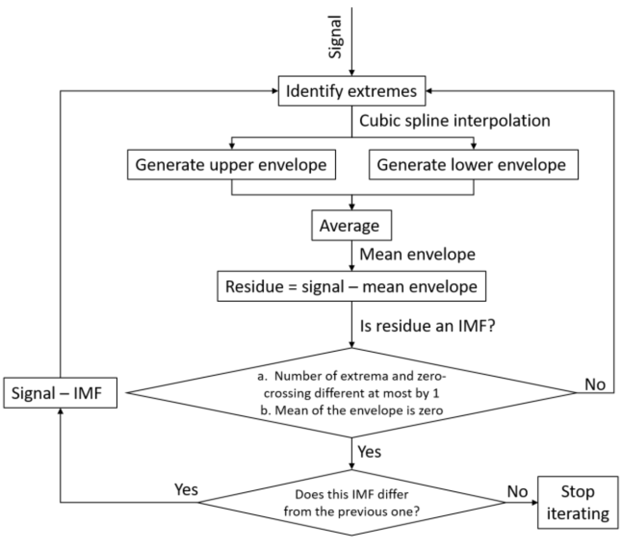

Figure 2. The flowchart of the EMD processing steps.

The exact process of the EMD computation is depicted in the flowchart of Fig. 2. First, the extreme points of the input signal are detected, then cubic spline interpolation is used to generate the upper and lower envelopes based on the maximum and minimum points, respectively. Next, the mean envelope is calculated from the upper and lower envelopes and subtracted from the original signal, creating a residual signal. This residual is regarded as a potential IMF. A proper IMF should satisfy two conditions; (i) the number of extreme points and the number of zero-crossings must be equal or the difference should not exceed one, (ii) the mean of the mean envelope should be approximately zero. In fact, it is very difficult for the residue to satisfy the two conditions at the same time, so standard deviation, SD, between two residues is usually used as the criterion for stopping the sifting process:

$$SD = \sum_{i=0}^{T} \frac{\left|R_{(k-1)}(t) - R_k(t)\right|^2}{(R_{(k-1)}(t))^2} \tag{1}$$

where $R_{k-1}$and R_k are the final residual signal in the sifting iteration $k-1$ and k, respectively. If the residue is an IMF then it will be compared with the IMF generated by the previous iteration. If they are the same, the process will be stopped, if not, the IMF will be subtracted from the input signal creating a new input signal for the next iteration round. After finishing the decomposition, the original signal can be represented as:

$$X(t) = \sum_{i=1}^{N} IMF_i(t) + R_N(t) \tag{2}$$

At any time point, each IMF represents a single instantaneous frequency value, i.e., each IMF describes a single oscillatory mode, consequently using the Hilbert transform analysis of the IMF, we can obtain the time-dependent instantaneous frequency of the given IMF.

Hilbert transform is a powerful tool for signal time-frequency analysis. The Hilbert spectrum can reflect the relationship between the instantaneous frequency of the signal and time, but due to the mixing of instantaneous information in non-stationary and nonlinear signals, the Hilbert transform cannot fully characterize its instantaneous phase and frequency features. The EMD method can stabilize the non-stationary signal by decomposing it into IMFs, therefore, the Hilbert transform (HT) of the IMFs can describe the time-frequency features of the signal more accurately:

$$HT(t) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{IMF(t')}{t - t'} dt' \tag{3}$$

where P indicates the Cauchy principal value. Using this formula, we can obtain the analytical signal $Z(t)$ of the IMF as:

$$Z(t) = IMF(t) + iHT(t) = a(t)e^{-\vartheta(t)} \tag{4}$$

where $\vartheta(t)$ is the instantaneous phase function. Based on the rate of change of the phase we can get the instantaneous frequency:

$$f(t) = \frac{d\vartheta(t)}{dt} \tag{5}$$

This method of combining EMD and Hilbert transform is called Hilbert-Huang transform (HHT).

An extension of the Hilbert-Huang transform is the Holo-Hilbert spectral analysis [8] that is able to capture the spectrum as well as the time-dependent spectrum changes for both additive and multiplicative signals, along with their inter and intra mode modulations. The detailed description of this method is beyond the scope of this paper.

IV. IMPROVEMENTS OF EMD

The EMD method, unfortunately, is not robust enough in the presence of noise. Moreover, intermittent noise can result in mode mixing, i.e., one IMF contains oscillations of largely disparate scales. Mode mixing will not only lead to wrong time-frequency distribution but also may make IMF lose its physical meaning.

In order to deal with the mode mixing problem, Wu *et al.* proposed a noise-assisted signal decomposition method in 2009 [9], the Ensemble Empirical Mode Decomposition (EEMD), shown in Fig. 3. Based on the uniform distribution of white noise spectrum, the algorithm adds white noise to the raw signal before decomposition, so that the intermittent noise is submerged in the added noise. Therefore, the distribution of extreme points of the signal will be more uniform, which can effectively suppress the mode mixing caused by intermittent factors. The EEMD method replicates the signal by adding to it random white noise. Then, these signals will be decomposed individually. The number of IMFs generated by each signal group may be inconsistent, which will lead to the inability to align each IMF during the averaging operation.

In order to solve the IMF alignment problem and improve the decomposition efficiency, Torres *et al.* proposed the complete EEMD with adaptive noise (CEEMDAN) in 2011 [10]. Compared with the EEMD method, CEEMDAN first adds noise to the raw signal to create the set of input signals, then calculates only the first IMF corresponding to each group, and averages these IMFs to obtain the first real IMF. Next, the real IMF will be subtracted from the raw signal and the next iteration is started. Fig. 4 shows the flow of one iteration. Since the averaging operation is performed after the first IMF of signals with white noise is calculated in each iteration, CEEMDAN solves the IMF alignment problem, and by controlling the parameters of the white noise added in different iterations, faster iterations can be achieved. However, the CEEMDAN method is still a decomposition method for single-channel signals. Although it solves the alignment problem of multiple groups of IMFs obtained from single-channel signals with different noise, it cannot solve the alignment problem of IMFs obtained by multi-channel decomposition.

Rehman *et al.* proposed the multivariate empirical mode decomposition (MEMD) method in 2010 [11,12] to solve the alignment problem of multiple IMFs obtained from multi-channel signal decomposition. As shown in Fig. 5, the MEMD method regards the input N-channel signal as a curve in $(N + 1)$ dimensional space, where the extra dimension is the time, and then sets a uniformly distributed set of direction vectors in the N-dimensional space. Curves are projected onto planes composed of different direction vectors and time axes to generate upper and lower envelopes of the projected signal on each direction vector, and the mean envelope is calculated from the envelopes along all directions, next comes the process of iteration and sifting. Although MEMD solves the problem of IMF alignment of multi-channel signals, it still suffers from the problem of mode mixing that is present in the one-dimensional EMD method.

As an improvement, Rehman *et al.* proposed the noise-assisted MEMD (NA-MEMD) method [13] to solve the mode mixing problem in MEMD. NA-MEMD does not add noise to the original multi-channel signal directly, but adds noise to the (N+1)-dimensional curve composed of the multi-channel signals, so it adds several noise channels and performs MEMD on the new multi-channel signals composed of the original multi-channel signals and the noise channels. Among all the IMFs obtained by decomposition, the IMFs corresponding to signal channels

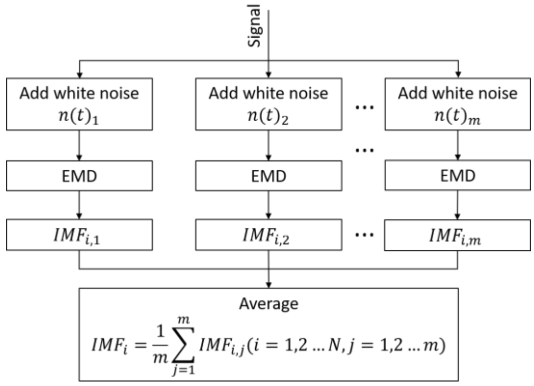

Figure 3. High-level structure of the EEMD calculation process

can be obtained by eliminating the IMFs corresponding to the noise channels.

EEG signals are generally multi-channel signals measured by multiple electrodes. Also, the signal of each channel is non-stationary, non-periodic and nonlinear, with certain level of correlations among channels. Since the MEMD method solves the alignment problem of IMFs, it can also preserve the mutual information among channels on the basis of analyzing non-stationary and nonlinear signals.

V. EMD APPLICATIONS IN EEG ANALYSIS

Although Empirical Mode Decomposition is still a relatively new method, several examples can be found in the literature for its application for EEG signal processing. In this section we present an overview of the potential areas that can benefit from EMD based on a review of the literature.

A. Pre-processing

Pre-processing is a mandatory step in any EEG analysis process. Raw measurement signals contain a significant amount of noise and unwanted artifacts that should be removed before further processing steps. Common examples are power line noise (50 or 60Hz), occular (eye movement of blink), heart (ECG or pulse) artifacts and muscle noise. Digital filtering can be used to remove DC or ultra-slow and high frequency components. Power line noise can be removed with a narrow notch filter. However, filtering can distort the amplitude and phase characteristics of the signal, therefore should be used with caution [14]. The gold standard for artifact removal is based on Independent Component Analysis (ICA) [15]. ICA can decompose a mixed signal into its statistically independent unmixed sources, hence artifact components can be identified, then eliminated from the component set. The inverse transform generates cleaned signals. It has been shown [16] that the separation of artifactual and neural signals is not perfect, therefore the removal of artifact components removes neural information too, and distorts the EEG signal. EMD can be used to selectively clean the artifact independent component instead of completely removing it [17][18]. DC and slow frequency component removal is also a crucial problem in EEG pre-processing. DC coupled EEG signals must have its DC offset removed before further processing. This offset, however, is not constant but a time-varying value depending on skin resistance changes, breathing and electrode conductivity variations. High-pass filtering can produce unwanted spectrum distortion if too high cut-off frequency or not steep enough slope is used, or waveform distortion (ringing) when filter slope is too steep. Zeng and Xu [19] provide an example how EMD can be used for detrending (removing the residual and last IMFs) instead of filtering. Similarly, power line noise and muscle artifacts can also be removed by eliminating 50/60Hz and high/frequency burst IMFs [20,21]. Santillan-Guzman *et al.* [22] propose an online EMD based preprocessing method that can be used in real-time settings.

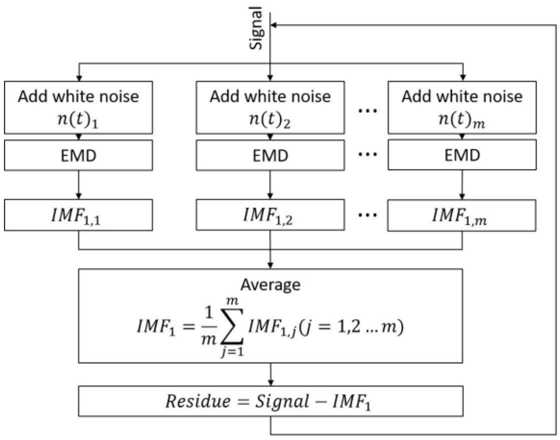

Figure 4. High-level structure of the CEEMDAN calculation process

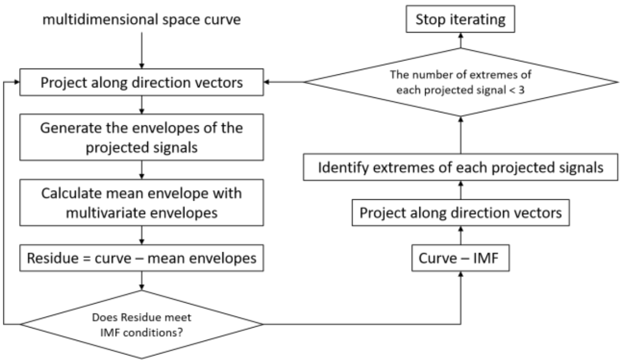

Figure 5. The flowchart of the MEMD processing steps

B. Cognitive Processing

Time-frequency analysis has become the de-facto standard in analyzing human task execution with EEG technology. The high temporal resolution allows tracking of subprocesses of task execution at millisecond steps. The Short-time Fourier transform or Wavelet transforms are used to extract time-varying spectral information. Since EMD generates narrow band IMFs representing distinct oscillations along with instantaneous frequency and phase information, it can be ideally used for characterizing time-varying frequency changes, amplitude and frequency modulations and phase coupling, providing a much more accurate and detailed view of the underlying neural processes. The instantaneous phase information can also be used to calculate time-varying phase locking value (PLV) to construct dynamic connectivity networks.

Nguyen et al [23] investigated the response of the visual system to amplitude modulated flicker generated steady-state visual evoked potentials (SSVEP). They showed that EMD - unlike Fourier transform based analysis - could correctly detect the carrier and modulating signals. In addition, they identified several new modulated carrier frequencies indicating the complexity of human visual processing. Alegre-Cortes *et al.* [24] demonstrate the advantages of the NA-MEMD method for detecting dynamic cortical oscillatory activity previously undetectable with Fourier-based methods. In an animal visual attention study, Liang *et al*. – using EMD –, were able to identify high and low frequency components in V4 area field potential measurements that distinguish between attending and not attending the same visual stimulus [25].

Tanaka *et al*. [26] compared wavelet and bivariate EMD approaches in detecting phase locking values (PLV) during a Dynamical Dot Quartet discrimination task. The finding is that EMD is more suitable for detection than the wavelet based method. Lee *et al.* [27] and Sweeney-Reed and Nasuto [28] also used EMD for detecting phase synchronization that can be used for connectivity network construction or cross frequency coupling computations.

Kawaguchi and T. Kobayashi [29] used MEMD and Hilbert spectrum analysis to detect voluntary rhythmic wrist activities. Compared to Fourier and wavelet methods, the MEMD approach could detect Event Related Synchronization and Desynchronization more accurately in addition to intrawave frequency modulation in the alpha band. Similar results are reported by Sweeney-Reed and Nasuto in [30] for finger tapping activity analysis.

Beta band activities and oscillations were investigated with EMD methods by Yeh et al [31] and Chang et al [32], while Glomb et al [33] and Javed et al [34] investigated non-task based, resting state activities. For clinical applications of EMD consult the review of Sweeney-Reed *et al* [35].

C. Parallel Implementation of EMD

Empirical Mode Decomposition has two major drawbacks that hinders its more widespread use. One is the lack of a formal mathematical foundation for the method, and the other one is its computational cost. The former will require extensive research in the years to come, but the computational aspects can be alleviated immediately by the use of massively parallel GPU algorithms. The most time consuming part of the EMD algorithm is the spline interpolation step, which must be executed thousands of times in ensemble or noise-assisted versions.

Several researchers reported parallel GPU implementations of different variants of the EMD method. Huang *et al.* present a GPU implementation of the original EMD algorithm [36]. The implementation consists of three main stages: extrema extraction, cubic-spline interpolation and IMF-candidate generation. Cubic spline interpolation relies on solving a system of tridiagonal equations which is an inherently sequential algorithm. Here the method of choice is the merging cubic spline interpolation that constructs the final spline from several smaller ones computed in parallel. The final speedup achieved with this method is 33.7x. Waskito *et al* [37] also implemented a GPU EMD version. Their version uses the Parallel Cyclic Reduction algorithm [38] for solving the tridiagonal system in parallel and achieved a 29x speedup.

Chen *et al* [39] describe a GPU implementation of the EEMD algorithm used for anesthesia monitoring. Here the major benefit is the large number of epochs (as ensembles are generated from epochs by adding white noise) that can be executed in parallel. Each epoch is parallelised in a fashion similar to the ones used for EMD above. The achieved speedup is above 100x. Finally, Chang et al [40] Mujahid *et al* [41] report on the GPU implementations of the MEMD method. Lang *et al.* [42] present a new fast algorithm that can improve the efficiency of the MEMD algorithm both in sequential and parallel implementations.

VI. Conclusion

This paper introduced Empirical Mode Decomposition, a promising method for the study and analysis of non-stationary and non-periodic signals, as well as its variants (EEMD, MEMD) that can be used effectively in EEG signal analysis. We have also summarized the advantages and shortcomings of the main EMD-based methods. Without completeness, we highlighted notable novel examples of using EMD for EEG signal processing and analysis. We are confident that there are many other exciting future application opportunities for the EMD method and its variants in EEG analysis, but the practical applicability of these methods mandates very fast implementations. In our future work, we will examine existing GPU implementations, and perform extensive performance evaluation and tuning to create new EMD-based methods and high-performance GPU implementations.

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