Integrated Path Tracking, and Control of a Fixed Wing UAV based on Dual Quaternion Parameterized Dynamics

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***Abstract*—Based on the dual quaternion formalism, the dynamics of a fixed wing UAV are formulated for path tracking, by incorporating the nonlinear gyroscopic terms. A control strategy is developed by exploiting the logarithmic map of the unit dual quaternion so as to ensure asymptotic stability in the three-dimensional space. The control and stabilization of the UAV, is carried out using dual quaternion parameterized dynamics such that a state feedback control law comprising of a PD controller ensures the simultaneous attitude and position tracking. Simulation results are presented to illustrate the applicability of this integrated approach in the tracking of a three-directional path.**

***Index Terms*—Dual quaternion, path tracking, attitude and position control, PD controller, YAK-54 UAV**

I. Introduction

The motion of a rigid body usually consists of both rotation and translation. The use of quaternions in modelling of the rotational kinematics is popular since it allows the dynamics of a rigid body to be completely defined in the 3D space while providing a double covering of the $SO(3)$ configuration space, and they are easy to deal with. Other methods also exist that can be used for modelling of rotational kinematics including Euler angles, Rodrigues parameters and transformation matrices, each having its' own advantages. The quaternion formulation has been used in rigid bodies [1], [2], satellites [3], quadrotor [4] and fixed wing [5], [7] unmanned aerial vehicles (UAVs), for attitude representation, control and modelling. For instance, in [5], a controller based on quaternion formulation was compared to a classical Euler-based attitude controller for attitude control of a highly maneuverable fixed wing aircraft. Repeated tracking of agile path maneuvers by a pilot can be challenging, and oftenly sub-optimal due to the unstable behaviour of the UAVs. Nevertheless, by using a quaternion-based attitude tracker designed in [8] to track agile maneuvers in a small fixed wing UAV, it was shown that the maneuver tracking could repeated severally with considerable precision.

However, since quaternion formulation only models the rotation motion, another formulation is required to handle the translation kinematics, i.e. the rotation and translation motions are handled independently. This is not desirable due to the natural coupling between these two motions. A dual quaternion which borrows heavily from dual numbers, consists of a unit quaternion and a translation transformation based on the unit quaternion, and can therefore be configured to track these two motions simultaneously. Hence, the rotation and translation motions are represented using a single quantity, a unit dual quaternion [9]. This representation is compact [10], easier and convenient to use, and the algebra involved turns out to be simpler by allowing straightforward algebraic manipulations.

Dual quaternions have been used to represent rigid body dynamics [11], robotics systems [12], satellite systems [13], unmanned aerial vehicles and spacecraft system dynamics [14], [15]. A PID controller was applied in [16] for trajectory tracking in a tilt-rotor UAV parameterized using dual quaternion-based system dynamics. A hybrid PID controller based on dual quaternion for pose tracking control was proposed in [17] where an integral term was introduced to compensate for constant disturbances. It was also shown that the hysteretic switching of a hybrid controller could avoid unwinding caused by the topological constraints. Sliding mode control were proposed for spacecrafts in [18], and for rigid body in [19] for the coordinated position and attitude control based on dual quaternion formalization. Other recent applications include in the control of a micro robot [20], rigid body [21], and quadrotor UAVs [16], [22]. In the literature of [23] and [13], it was shown that asymptotic convergence of the tracking error, and stability of the closed loop dynamics parameterized using dual quaternions are guaranteed for continuous feedback control laws.

In this paper, path tracking of a fixed wing UAV based on dual quaternion parameterized kinematics for the integrated tracking of both the rotational and translational motions is presented. In contrast to [24], [25], where the gyroscopic terms in the Newton's equation are neglected, in this work the design takes into account the contribution of these terms as provided for in [26] since they are important especially when dealing with agile maneuvers. The dynamic and kinematic equations of motion are presented, and used in the path tracking simulations where a PD feedback controller is designed using the dual quaternion error, composed of the integrated attitude and position error. The rest of the paper is organized as follows: Section II briefly introduces the quaternion and dual quaternion principles, and the related algebra while Section III provides the rigid body dynamics and kinematics. Section IV presents the PD controller by exploiting the dual quaternion logarithmic mapping. Section V gives the simulation results and discussions, and the conclusions are given in Section VI.

II. Mathematical Preliminaries

A. Quaternions

Quaternions are also known as hyper complex numbers since they can be represented by one real and three imaginary numbers as q$\triangleq$ $q_s + q_1\hat{i} + q_2\hat{j} + q_3\hat{k}$ where , such that $\hat{i}^2 = \hat{j}^2 = \hat{k}^2 = \hat{i}\hat{j}\hat{k} = -1$. An alternative representation is given by $q = [\eta, \varepsilon]^T$ where $\eta \in \mathbb{R}^1$ and $\varepsilon = [q_1, q_2, q_3] \in \mathbb{R}^3$ are the scalar and vector parts respectively, such that the product space $q \in \mathbb{R}^1 \times \mathbb{R}^3 \cong \mathbb{R}^4$. A unit quaternion can then be defined as

$$q = \left[\cos\left(\frac{\theta}{2}\right), \sin\left(\frac{\theta}{2}\right)\mathbf{n}\right]^T.$$

where θ is a rotation angle and n is a three-dimensional vector representing the axes. The conjugate of a quaternion is given as

$$q^* = q^{-1} = [\eta, -\varepsilon^T]^T.$$

The product of two quaternions is given as $q_1 \cdot q_2$. The logarithm of a quaternion is $\ln(q) = [0, \frac{\theta}{2}\mathbf{n}^T]^T \cong \frac{\theta}{2}\mathbf{n}$, for $0 \leq \theta < 2\pi$. Other basic operations involving quaternions can be found in [27]. The time derivative of a quaternion is dependent on the angular velocity vector and is shown in (1):

$$\dot{q} = \frac{1}{2}\begin{bmatrix} 0 & -\omega^T \\ \omega & -\mathbf{S}(\omega) \end{bmatrix} q = \frac{1}{2}T(q)\bar{\omega} \tag{1}$$

where $T(q) = \begin{bmatrix} \eta & -\varepsilon^T \\ \varepsilon & \eta\mathrm{I} + S(\varepsilon) \end{bmatrix}$ and $\bar{\omega} = \begin{bmatrix} 0 \\ \omega \end{bmatrix}$. The quantity $\mathbf{S}(\cdot)$ is a vector cross-product operator such that $\mathbf{S}(v) = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}$, for a vector $v = (v_1, v_2, v_3)^T$.

B. Dual Quaternions

A dual quaternion is defined as

$$\hat{q} = q_r + \epsilon q_d$$

where $q_r \in \mathbb{R}^4$ and $q_d \in \mathbb{R}^4$ are the real part and dual part of the dual quaternion respectively, such that q_r is a unit quaternion and q_d is the translation transformation. The dual operator ϵ, is defined as $\epsilon^2 = 0$ such that $\epsilon \neq 0$. Using this analogy, a 3D motion consisting of a rotation $q := R \in \mathbb{R}^{3\times3}$, followed by a translation $\vec{p} \in \mathbb{R}^3$ can be represented using a dual quaternion as:

$$\hat{q} = q_r + \frac{\epsilon}{2} q \cdot \vec{p} \tag{2}$$

Equation (2) can be represented as

$$\hat{q} = \left[\cos\frac{\hat{\theta}}{2}, \sin\frac{\hat{\theta}}{2}\hat{\mathbf{n}}\right]$$

where $\hat{\theta} = \theta + \epsilon d$ is the dual angle about the screw axis $\hat{\mathbf{n}}$, see [9]. The product of two dual quaternions is expressed as $\hat{q}_1 \circ \hat{q}_2$. The logarithmic mapping of a unit dual quaternion can be defined as $\ln\hat{q} = \frac{1}{2}(\theta + \epsilon\vec{p})$, where θ is a rotation angle of q_r. The norm of the dual quaternion $\hat{q}$ is given by $\|\hat{q}\| = \sqrt{\hat{q} \circ \hat{q}^*}$, where the dual quaternion conjugate $\hat{q}^* = q_r^* + \epsilon q_d^*$. The difference between two dual quaternions, $\hat{q}_a$, and $\hat{q}_b$ is expressed as

$$\hat{q}_{ab} = \hat{q}_a^* \circ \hat{q}_b. \tag{3}$$

For a unit dual quaternion, the relationships $q_r \cdot q_r = \mathrm{I}$, and $q_r \cdot q_d = 0$ must be satisfied at all times. The time derivative of a unit dual quaternion (2) is given as

$$\dot{\hat{q}} = \frac{1}{2}\hat{q} \circ \xi \tag{4}$$

where ξ is known as a twist, and is expressed as $\xi = \omega + \epsilon(\dot{\vec{p}} + \omega \times \vec{p})$. Other basic operations of dual quaternions follow closely those of dual numbers, see [28].

III. UAV Mathematical Model

Due to the advantages of using a quaternion, q in attitude representation, and the integral role they can play in tracking the orientation of a flying vehicle for a complete flight envelope, we employ them to model the dynamics of a UAV. The general equations of motion describing the dynamics, and orientation of a UAV in the body frame can be written as:

$$\dot{V}_b = \frac{1}{m}F_b - \omega_b \times V_b \tag{5}$$

$$\dot{\omega}_b = J^{-1}\left[M_b - \omega_b \times (J\omega_b)\right] \tag{6}$$

$$\dot{q} = \frac{1}{2}q \otimes \omega_b \tag{7}$$

where the translational velocity $V_b = [u, v, w]^T$, angular velocity $\omega_b = [p, q, r]^T$, and J is the inertia matrix. The forces, F_b and moments, M_b act on the aircraft, and are resolved in the body frame.

Equation (7) can be represented in the form of (1) with, $\bar{\omega} = [0, \omega_b]^T$. The forces, and moments in the three coordinate axes are represented as in (8) and (9) respectively.

$$\frac{F_b}{\bar{q}S} = F^{aero} + F^{grav} + F^{thr} = \begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix} + g_w + \begin{bmatrix} thr \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

$$\frac{M_b}{\bar{q}S} = \begin{bmatrix} b\left(C_{l\beta}\beta + C_{lp}\frac{b}{2V_a}p + C_{lr}\frac{b}{2V_a}r + C_{l\delta a}\delta_a + C_{l\delta r}\delta_r\right) \\ \bar{c}\left(C_{m\alpha}\alpha + C_{mq}\frac{\bar{c}}{2V_a}q + C_{m\delta e}\delta_e\right) \\ b\left(C_{n\beta}\beta + C_{np}\frac{b}{2V_a}p + C_{nr}\frac{b}{2V_a}r + C_{n\delta a}\delta_a + C_{n\delta r}\delta_r\right) \end{bmatrix} \tag{9}$$

The various variables are described as $(C_x, C_y, C_z)^T = R_b^s(-C_D, C_Y, -C_L)^T$, $C_D = C_{D0} + {C_L}^2/\pi A_r K_{Osw}$, $C_Y = C_{y\beta}\beta + C_{yp}\frac{b}{2V_a}p + C_{yr}\frac{b}{2V_a}r + C_{y\delta a}\delta a + C_{y\delta r}\delta_r$, $C_L = C_{L0} + C_{L\alpha}\alpha$, $g_w = mg[-\sin\theta, \sin\phi\cos\theta, \cos\phi\cos\theta]^T$, thr is the thrust, and R_b^s is a rotation matrix from the body frame to stability frame. The dynamic pressure $\bar{q} = \frac{1}{2}\rho {V_a}^2$, where ρ is the air density, S is the wing area, b is the wing span, and

$\bar{c}$ is the mean aerodynamic chord. The velocity, V_a, sideslip angle, β and angle of attack, α which are derived components of V_b, are computed as:

$$\begin{aligned}
\dot{V}_a &= -\frac{\bar{q}S}{m}C_{D_W} + \frac{thr}{m}C_\alpha C_\beta + g_w(1) \\
\dot{\beta} &= \frac{\bar{q}S}{mV_a}C_{Y_W} + \Gamma_1 - \frac{thr}{mV_a}C_\alpha S_\beta + \frac{g_w(2)}{V_a} \\
\dot{\alpha} &= -\frac{\bar{q}S}{mV_a C_\beta}C_{LW} + q - \Gamma_2 - \frac{thr}{mV_a C_\beta}S_\alpha + \frac{g_w(3)}{V_a C_\beta}
\end{aligned}$$

where $C_{D_W} = C_D C_\beta - C_Y S_\beta$, $C_{Y_W} = C_Y C_\beta + C_D S_\beta$, $C_{LW} = C_L + C_{Lq}\frac{\bar{c}}{2V_a}q + C_{L\delta e}\delta_e$, $\Gamma_1 = pS_\alpha - rC_\alpha$, $\Gamma_2 = T_\beta(pC_\alpha + rS_\alpha)$. The $C_{(\cdot)}$ are the aerodynamic stability and control derivatives, and C_α, S_β represent the trigonometry functions of the respective subscript variables. Equation (6) can then be expressed as:

$$J\dot{\omega}_b = -\mathbf{S}(\omega_b)J\omega_b + f(x) + D(x)\omega_b + G(x)u \tag{10}$$

where $x = (V_a, \alpha, \beta)$, and u represent the control deflections $[\delta_a, \delta_e, \delta_r]^T$. The composition of the components in (10) is:

$$f(x) = \bar{q}S \begin{bmatrix} b(C_{l_0} + C_{l_\beta}\beta) \\ \bar{c}(C_m + C_{m_\alpha}\alpha) \\ b(C_{n_0} + C_{n_\beta}\beta) \end{bmatrix} \tag{11}$$

$$D(x) = \bar{q}S \begin{bmatrix} \frac{b^2}{2V_a}C_{l_p} & 0 & \frac{b^2}{2V_a}C_{l_r} \\ 0 & \frac{\bar{c}^2}{2V_a}C_{m_q} & 0 \\ \frac{b^2}{2V_a}C_{n_p} & 0 & \frac{b^2}{2V_a}C_{n_r} \end{bmatrix} \tag{12}$$

$$G(x) = \bar{q}S \begin{bmatrix} bC_{l_{\delta_a}} & 0 & bC_{l_{\delta_r}} \\ 0 & \bar{c}C_{m_{\delta_e}} & 0 \\ bC_{n_{\delta_a}} & 0 & bC_{n_{\delta_r}} \end{bmatrix} \tag{13}$$

With this breakdown, a set of control deflections u can be designed to achieve tracking of a desired orientation q in (7). With some re-organization of (5) and (8), it is realized that the thrust force can be used as the design variable for the translation dynamics (5), where the Euler angles in (8) are given as:

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \text{atan2}\left(2(q_s q_1 + q_2 q_3), q_s^2 + q_3^2 - q_1^2 - q_2^2\right) \\ \text{asin}\left(2(q_s q_2 - q_1 q_3)\right) \\ \text{atan2}\left(2(q_s q_3 + q_1 q_2), q_s^2 + q_1^2 - q_2^2 - q_3^2\right) \end{bmatrix}$$

IV. Controller Design

A. Dual Quaternion Formulation

The translation and rotational equations (5-6) representing the dynamics of a rigid body in the body coordinate frame can be rewritten as:

$$\begin{cases} m(\ddot{p}_b + \omega_b \times \dot{p}_b) = f \\ J\dot{\omega}_b + \omega_b \times J\omega_b = \tau \end{cases} \tag{14}$$

where m is the mass of the vehicle, p_b is the position vector in the body frame, f are forces acting on the body, ω_b is the angular velocity, J is the inertia matrix, and τ are the control torques. Using (4), the rotational kinematics of a rigid body expressed using a unit dual quaternion is given as

$$\dot{\hat{q}} = \frac{1}{2}\hat{q} \circ \xi_b \tag{15}$$

where $\xi_b = \omega_b + \epsilon(\dot{p}_b + \omega_b \times p_b)$. Differentiating ξ_b with respect to time results to

$$\dot{\xi}_b = \dot{\omega}_b + \epsilon(\ddot{p}_b + \dot{\omega}_b \times p_b + \omega_b \times \dot{p}_b). \tag{16}$$

Substituting the second relation of (14) into (16), and carrying out some rearrangements, results into (17),

$$\dot{\xi}_b = \hat{F} + \hat{U} \tag{17}$$

where $\hat{F} = a + \epsilon(a \times p_b + \omega_b \times \dot{p}_b)$, $\hat{U} = J^{-1}\tau + \epsilon(f/m + J^{-1}\tau \times p_b)$ and $a = -J^{-1}\omega_b \times J\omega_b$. Equations (16)-(19) form the system dynamics parameterized using the unit dual quaternion.

B. Error Dynamics

Let $q_d, p_d, \omega_d, \dot{\omega}_d$ be the desired attitude, position, angular velocity, and acceleration respectively. The desired configuration using dual quaternion formalization $\hat{q}_d$, is $\hat{q}_d = q_d + \frac{\epsilon}{2}q_d \cdot p_d$. Therefore, the tracking error between the current configuration, $\hat{q}$ and a desired configuration, $\hat{q}_d$ is given as

$$\hat{q}_e = \hat{q}_d^* \circ \hat{q}. \tag{18}$$

Equation (18) can be written in the form of (2) as

$$\hat{q}_e = q_e + \frac{1}{2}q_e \circ p_b^e \tag{19}$$

where $q_e = q_d^* \cdot q$ is the quaternion error [29] and $p_b^e = p_b - Ad_{q_e^*}p_d$. Ad_* represents the adjoint transformation of a vector i.e. $Ad_q v = q \cdot v \cdot q^*$ for a vector v. The error dynamics can thus be expressed as:

$$\dot{\hat{q}}_e = \frac{1}{2}\hat{q}_e \circ \xi_b^e \tag{20}$$

where $\xi_b^e = \xi_b - Ad_{\hat{q}_e^*}\xi_d$ and $Ad_{\hat{q}_e^*}\xi_d = \hat{q}_e^* \circ \xi_d \circ \hat{q}_e$. Taking the time derivative of ξ_b^e, and making some algebraic manipulations see [14] [16], the expression in (21) is obtained.

$$\dot{\xi}_b^e = \dot{\xi}_b - Ad_{\hat{q}_e^*}\xi_d - \left[\hat{0}, Ad_{\hat{q}_e^*}\xi_d \times \xi_b^e\right] \tag{21}$$

Substituting (17) into (21), results into

$$\dot{\xi}_b^e = \hat{F} + \hat{U} - Ad_{\hat{q}_e^*}\xi_d - \left[\hat{0}, Ad_{\hat{q}_e^*}\xi_d \times \xi_b^e\right] \tag{22}$$

C. Controller

The objective is to design a control law to asymptotically track the desired target, that is $\hat{q}$ should converge to $\hat{q}_d$ asymptotically as $t \to \infty$. Similarly, ω_b should converge to ω_d that is, $\xi_b^e \to \hat{0}$ as $t \to \infty$. If this is achieved, then it implies that (q, p_b) simultaneously converges to (q_d, p_d) respectively. The PD controller described in [22], which is formulated based on the logarithmic mapping of the dual quaternion Lie-algebra is used. The control law is given as in (23), which includes a feedforward compensation.

$$\hat{U} = -2\hat{k}_p \ln \hat{q}_e - \hat{k}_v \xi_b^e - \hat{F} + Ad_{\hat{q}_e^*}\xi_d + Ad_{\hat{q}_e^*}\xi_d \times \xi_b^e \tag{23}$$

The gains $(\hat{k}_p, \hat{k}_v) > 0$ are as $\hat{k}_p = k_p + \epsilon k_p$ and $\hat{k}_v = k_v + \epsilon k_v$ such that similar gains are applied to the real, and dual parts.

From the relations subsequent to (19),

$$\hat{U} = J^{-1}\tau + \epsilon(f/m + J^{-1}\tau \times p_b) \tag{24}$$

which is in dual vector format, and can be rewritten as $\hat{U} = r(\hat{U}) + \epsilon d(\hat{U})$, where $r(\hat{U}) = J^{-1}\tau$, and $d(\hat{U}) = (f/m + r(\hat{U}) \times p_b)$. Rearranging the terms reveals that:

$$f = m\left(d(\hat{U}) - r(\hat{U}) \times p_b\right)$$
$$\tau = \left(Jr(\hat{U})\right)$$

From τ, $\dot{\omega}_b$ can be extracted from (16), and consequently the control deflections u, in (10). Moreover, from f, then $\ddot{p}_b$ can be extracted from (14), and subsequently $\dot{p}_b$ and p_b through numerical integration.

V. Simulations

The unmanned aerial vehicle used in this experiment is YAK-54 UAV whose physical properties are given in [30], and the aerodynamic coefficients are given in Table 1. The mass of the vehicle is $12.755kg$, the Oswald constant, K_{Osw} was taken as 0.85, and the maximum thrust was calculated to be about $150N$. The initial values of the

TABLE I
YAK-54 UAV aerodynamic coefficients

Coeff.	Value	Coeff.	Value
C_{D0}	0.0526	C_{D_α}	-0.0863
C_{Y_β}	-0.3462	C_{Y_p}	0.0073
C_{Y_r}	0.2372	$C_{Y_{\delta_r}}$	0.1928
C_{L0}	0.1470	C_{L_α}	4.5363
C_{L_q}	5.1515	$C_{L_{\delta_e}}$	0.3762
C_{l_β}	-0.0255	C_{l_p}	-0.3817
C_{l_r}	0.0504	$C_{l_{\delta_a}}$	0.3490
$C_{l_{\delta_r}}$	0.0154	C_{m0}	-0.0018
C_{m_α}	-0.3701	C_{m_q}	-8.5026
$C_{m_{\delta_e}}$	-0.8778	C_{n_β}	0.0954
C_{n_p}	-0.0156	C_{n_r}	-0.1161
$C_{n_{\delta_a}}$	-0.0088	$C_{n_{\delta_r}}$	-0.0996

UAV were set as $q_{nb}(0) = [0.9932, 0.0324, 0.0626, 0.0929]^T$, $\omega_{nb}(0) = [0.01, 0.01, 0.01]^T rad/s$, $p_b = [1, 1, -99]^T m$, $\dot{p}_b = [30, 0, 0]^T m/s$ for the attitude, angular velocity, position and velocity respectively. The velocity was to be maintained at $30m/s$ throughout the path tracking. The gains k for the controller were set as $k_p = 10$ and $k_v = 24$ respectively.

Applying the properties of YAK-54 UAV in the dynamic model presented in Section III, and the controller discussed in Section IV, path tracking is investigated for the path described in [31]. The path consists of different motions including a straight-line section, a spiral section and a half circle turn in such a manner that there is smooth transitions between the sections.

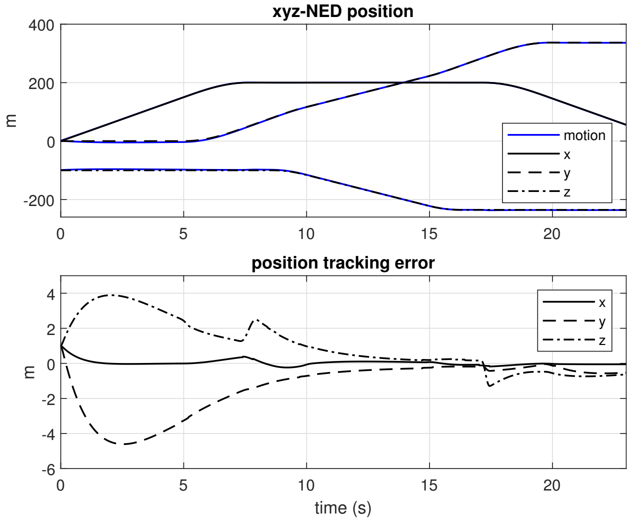

Fig. 1. UAV position during path tracking, and tracking errors

The tracking of the relative positions of the path is shown in Fig. 1 and the associated 3D motion is shown in Fig. 2. Due to the initial position and attitude values, and their coupling, the position tracking errors are large during the initial transient stage. Thereafter, the tracking errors lie to within acceptable deviations along the path as shown in position error graph in Fig. 1. Moreover, due to the use of numeric integration, position errors can be propagated through the simulation.

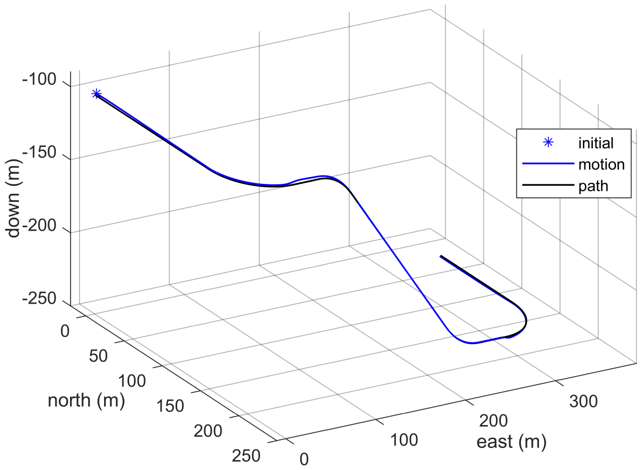

Fig. 2. Tracking of the 3D path

In Fig. 3, the evolution of the attitude trajectories using the quaternion, q_{nb} for the attitude tracking are shown. The corresponding orientation vis-à-vis the desired orientation using Euler angles are shown in Fig. 4. The initial errors in orientation are attributable to the controller adjusting the UAV from its initial pose to the desired pose, which too has an effect on the position tracking due to the dual quaternion coupling. The associated angular velocity components during tracking are shown in Fig. 5.

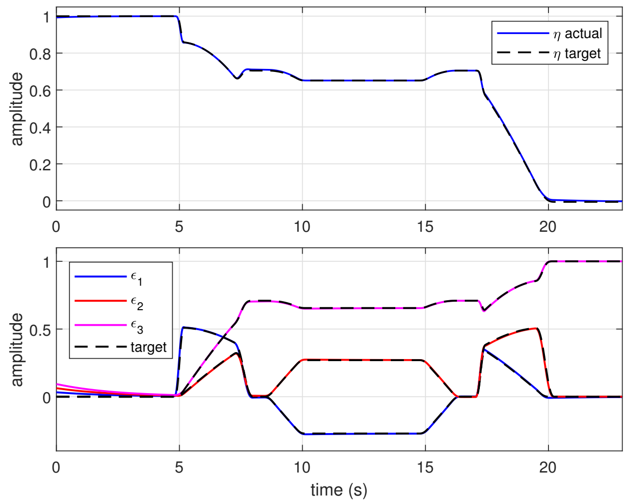

Fig. 3. Tracking and the evolution of the quaternion trajectory

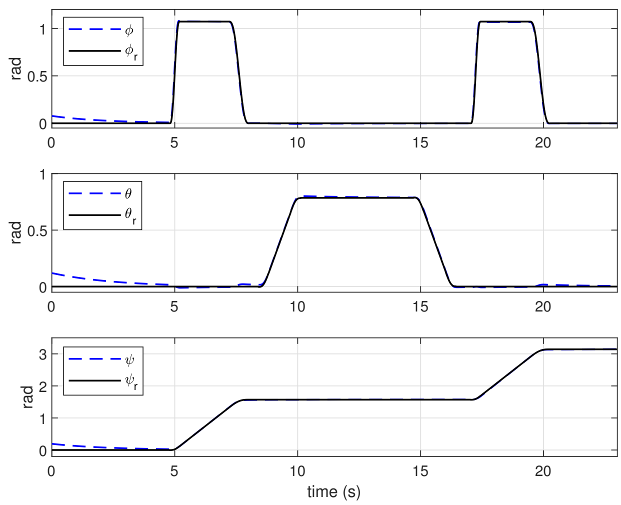

Fig. 4. Tracking of the attitude expressed in Euler angles

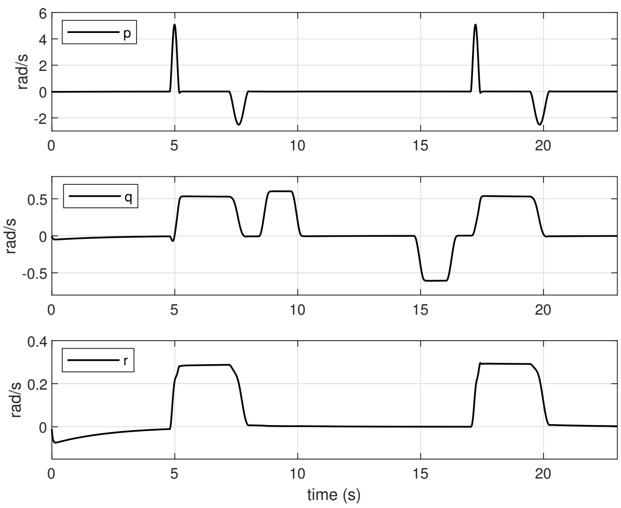

Fig. 5. Angular velocity components during tracking

The velocity through the path tracking is shown in Fig. 6, alongside the thrust force. The velocity is relatively constant at $30m/s$ as the thrust ensure that the UAV maintains a steady velocity while it tracks the desired path. From about 8-10 secs, the thrust increases steadily in response to the steep change in the z position, and later on returns to normal range after that maneuver has been cleared. The control deflections during the path tracking are shown in Fig. 7.

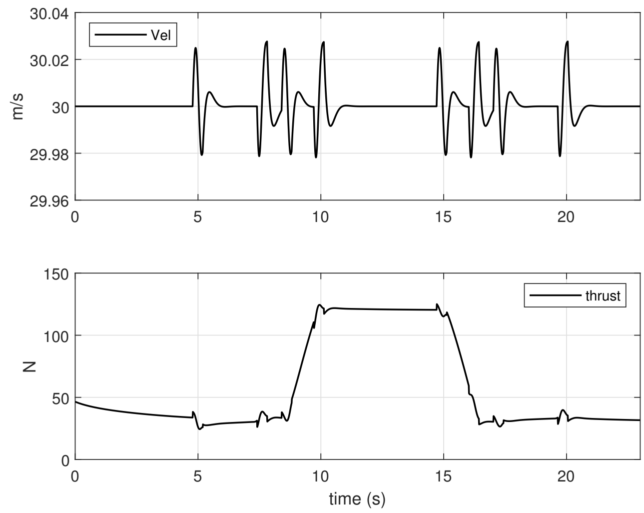

Fig. 6. Evolution of velocity, and the thrust force

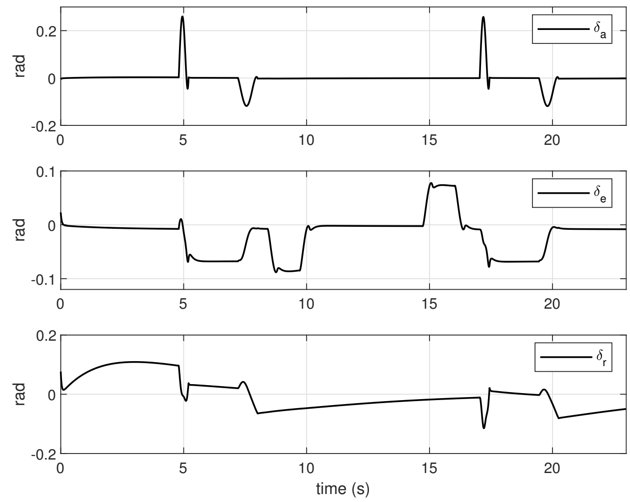

Fig. 7. Control signals during path tracking

VI. Conclusions

This paper has presented the integrated attitude and position tracking of a fixed wing UAV based on dual quaternion parameterized dynamics. A PD controller using simple state feedback while exploiting the mathematical simplicity and logarithmic mapping of a dual quaternion was used to achieve the asymptotic tracking. The formulation presented here does not cancel the nonlinear gyroscopic terms in the control dynamics, and thus a practical formulation with complete parameterization of the dynamics. The numerical simulation results presented have exemplified the applicability of this formalism to control and track a desired three directional path using a UAV.

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