Application of the Signal Samples Approximation for Accurate RMS Measurement

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Abstract **- Nowadays the root-mean-square (RMS) is one of the most informative parameters of electrical power grid signals. By definition, the RMS measurement technique involves averaging the square of the input signal over time multiple of the input signal period. However, for digital measurement methods, the input signal is represented as a sequence of discrete samples obtained by applying analog-to-digital converter (ADC). The paper discusses ways of approximating of the signal samples by applying polynomial functions of zero, first and second order. Analytical expressions are obtained for the RMS measurement error for the case of applying of different order approximation polynomials. The influence of the input signal amplitude, signal frequency, initial phase, sampling frequency and total measurement time on the RMS measurement error for the case of applying these measurement approaches is considered. The influence of the input signal frequency deviation on the RMS measurement error has been especially thoroughly investigated. The methods of reducing the RMS measurement error for the case of sinusoidal and polyharmonic input signals are proposed. By application of Matlab and Simulink software packages, a simulation mathematical model for all considered approaches is performed.**

Keywords – root mean square; simulation; approximation; measuement error; frequency deviation

I. Introduction

The root mean square (RMS) [1]-[3] is one of the most informative parameters of electrical power grid signals. This parameter is used for the case of performing indirect measurement of signal integral parameters (active, reactive, apparent power, frequency, harmonic distortion).

Digital measurement methods are currently applied to RMS measurement. The main feature of these methods is the need to process of the input signal discrete samples, therefore, the direct implementation of RMS determination is impossible. The most popular approach to digital RMS measurement is the so-called square averaging method [4]-[8]. This method is based on the approximation of discrete samples by applying a zero-order polynomial function. This method is characterized by simple implementation and relative simplicity of error analysis [4]-[8]. There are many techniques to reduce the measurement error. However, the application of these techniques is associated with a significant increase of the measurement time, which is unacceptable for a number of practical tasks.

The performed research [4]-[8] shows that in the case of RMS measurement of polyharmonic signals, the application of the method of averaging of the squares of samples results to a significant additional error. This disadvantage is especially relevant for the case of the RMS measurement of current, which is characterized by large harmonic coefficients values compared to voltage. In addition to the method based on averaging of the squares of samples, there are low-pass filtration [9]-[10] and spectral analysis [11]-[12] methods. Both methods make it possible to measure polyharmonic signals, but achieving high accuracy is associated either by an measurement time increase or by implementation of additional transducers. This significantly limits it practical application.

The main task of this work is a search a new algorithms of the RMS measurement, which make it possible to increase the measurement accuracy for both sinusoidal and polyharmonic signals. The paper will consider methods based on the approximation of discrete samples of the measured signal by first order (piecewise linear) and second order approximation polynomials.

II. Application of Approximation Polynomials for the RMS Measurement

A. The RMS Digital Measurement Concept

The root mean square (RMS) of the signal x is understood as a value equal to the square root of the mean value of the square signal over a time which is multiple of the measured signal period [1]-[3]:

$$X_{id} = \sqrt{\frac{1}{T_A}\int_0^{T_A} x^2(t)dt}\ , \qquad (1)$$

where $x(t)$ denotes measured signal; T_A denotes measurement time, multiple of the signal period; X_{id} denotes true RMS value.

For the case of digital measurement methods, the input signal $x(t)$ is represented as a sequence of discrete samples $x[n]$. For this reason, direct application of (1) is impossible. To apply (1), the discrete signal $x^2[n]$ must be restored to a continuous signal $x_1^2(t)$. Polynomials of the zero, first and second orders can be applied as a restoring function.

B. Application of the Zero-Order Polynomial

The simplest recovery function is a zero-order polynomial. In the case of applying such restoring function, the resulting signal changes its value only until the next sample is reached [4]-[8]:

$$x_1^2(t)=\begin{cases} x^2[n-1], & (n-1)T_S \le t < (n)T_S, \\ x^2[n], & nT_S \le t < (n+1)T_S; \end{cases} \tag{2}$$

where n denotes discrete sample number; T_S denotes sampling time.

In the case of using the approximating function (2), the RMS value takes the following form (see (1)):

$$X_{p0}=\sqrt{\frac{1}{N}\sum_{n=0}^{N-1}x^2[n]}\,, \tag{3}$$

where N denotes samples number during total measurement time T_A.

The number of samples N can be determined from the nominal or measured value of the input signal frequency. In the second case it is possible to achieve smaller values of the RMS measurement error (will be shown below):

$$N=round\left(\frac{T_A}{T_S}\right)=round\left(\frac{m\cdot T}{T_S}\right), \tag{4}$$

where T denotes measured or true value of signal period; m denotes number of observed signal periods over T_A.

C. Application of the First-Order Polynomial

In the case of applying a first-order polynomial, the dependence of the reconstructed signal $x_1^2(t)$ between samples $x^2[n]$ will be linear and the overall dependence of the signal $x_1^2(t)$ will be a piecewise linear function. In this case, the dependence $x_1^2(t)$ for each sampling step:

$$x_1^2(t)=\begin{cases} a_n\cdot t+b_n;\ n\cdot T_S < t < (n+1)\cdot T_S, \\ x^2[n] \quad for\ t=n\cdot T_S, \\ x^2[n+1] \quad for\ t=(n+1)\cdot T_S; \end{cases} \tag{5}$$

where a_n, b_n denotes coefficients of the piecewise linear approximating function for time interval $n\cdot T_S \div (n+1)\cdot T_S$.

The equation system (5) can be applied to obtain the values of the approximating coefficients:

$$a_n=\frac{x^2[n+1]-x^2[n]}{T_S}, \quad b_n=x^2[n]\cdot(n+1)-x^2[n+1]\cdot n. \tag{6}$$

Applying the obtained (5) and (6) to (1), we obtain an algorithm for RMS measurement in the case of piecewise linear approximation of the signal $x^2(t)$:

$$X_{p1}=\sqrt{\frac{1}{2N}\sum_{n=0}^{N-1}x^2[n]+\frac{1}{2N}\sum_{n=1}^{N}x^2[n]}\,, \tag{7}$$

When obtaining (7), the formula was used to calculate the area under the piecewise linear function. Considering that both sums differ only by one sample (first and last), then (7) can be converted to:

$$X_{p1}=\sqrt{\frac{1}{N}\sum_{n=0}^{N-1}x^2[n]+\frac{x^2[N]+x^2[0]}{2N}}\,. \tag{8}$$

The samples number N is also calculated according to (4). By comparing the algorithms for the stepwise and piecewise linear functions, it can be seen that these algorithms differ slightly of implementation complexity.

D. Application of the Second Order Polynomial

When applying a second-order approximating polynomial, the dependence of the signal $x_1^2(t)$ for each sampling step can be represented in the following form:

$$x_1^2(t)=\begin{cases} a_n\cdot t^2+b_n\cdot t+c_n;\ (n-1)\cdot T_S < t < (n+1)\cdot T_S, \\ x^2[n-1] \quad for\ t=(n-1)\cdot T_S, \\ x^2[n] \quad for\ t=n\cdot T_S, \\ x^2[n+1] \quad for\ t=(n+1)\cdot T_S; \end{cases} \tag{9}$$

By solving system (9), expressions can be obtained for the approximating polynomial coefficients a_n, b_n, c_n for the case of evenly spaced samples $x^2[n-1]$, $x^2[n]$, $x^2[n+1]$ with sampling step T_S:

$$a_n=\frac{x^2[n+1]+x^2[n-1]-2\cdot x^2[n]}{2\cdot T_S^2}, \quad b_n=\frac{x^2[n+1]-x^2[n-1]}{2\cdot T_S}, \quad c_n=x^2[n]. \tag{10}$$

Substituting the obtained (10) into the general algorithm of RMS measurement (1), we obtain an algorithm for RMS measurement when applying the second-order approximation polynomial:

$$X_{p2}=\sqrt{\sum_{n=2}^{N+1}\frac{x^2[2n-2]+4x^2[2n-1]+x^2[2n]}{6\cdot(N+1)}}\,. \tag{11}$$

For the convenience of further analysis, this algorithm can be represented as:

$$X_{p2}=\sqrt{\sum_{n=0}^{N-1}\frac{x^2[2n]+4x^2[2n+1]+x^2[2n+2]}{6\cdot(N+1)}}\,. \tag{12}$$

The number of samples and the sampling time when performing the approximation by a second-order polynomial are related by the following expression (in this expression it is taken into account that the samples number must be even):

$$T_A = T_S \cdot (2N+2). \tag{13}$$

As can be seen from the obtained expressions, an increase of the approximating polynomial order requires additional arithmetic operations. But since the (11)-(12) the area under the curve $x_1^2(t)$ is determined at once for two samplings, the increase in arithmetic operations for the total measurement time will not be significant.

III. The RMS Measurement Error for the Case of Application of the Zero-Order Approximation Polynomial

A. *The Case of Sinusoidal Signal*

For the case of a sinusoidal input signal, the RMS measurement error can be represented as [4]-[8]:

$$\delta_{p0} \cong -\frac{\cos((N-1)\omega T_S + 2\alpha)\sin(\omega N T_S)}{2N\sin(\omega T_S)}, \tag{14}$$

where α denotes input signal initial phase; ω denotes input signal angular frequency.

Fig. 1 shows the dependence of the RMS measurement error on the input sinusoidal signal frequency. The sampling rate is chosen equal to 10 kHz, the amplitude value is 1 V, the initial phase is chosen equal to 0 radian (red dependence), π/6 radian (blue dependence) and π/3 radian (green dependence). The measurement time is chosen equal to 0.2 sec (ten nominal periods of the input signal). It can be seen from the plots that the dependences characterizes a variable behavior and there are singular frequency points for which (4) is satisfied), for which the error is zero. The initial phase affects to the error dependence and results to its displacement relative to the zero level. There are special values of the initial phase for which the resulting error value is equal to zero.

In the case of an arbitrary initial phase (adjustment of the initial phase is not performed and the start of measurement and the input signal initial phase are not synchronized) the maximum error value takes the form:

$$\delta_{p0,\max} \cong -\frac{\sin(\omega N T_S)}{2N\sin(\omega T_S)}. \tag{15}$$

Fig. 2 shows the dependence of the RMS error on the signal initial phase. From the obtained dependence it is seen that the error takes zero values for certain values of the initial phase. When performing the figure, the relative frequency deviation from the nominal value was taken to be 0.01 %. Fig. 3 shows the dependence of the RMS measurement error on the total measurement time. It is seen that with increasing measurement time, the error tends to decrease. For this reason, the maximum measurement time should be chosen, which corresponds to (4). Fig. 4 shows the dependence of the maximum RMS measurement error on the arbitrary initial phase of the input signal. When performing (by Matlab software) Fig. 2, Fig. 3 and Fig. 4, the simulation parameters was selected equal to parameters which are applied for Fig. 1.

In practice, there are two main approaches to reduce the RMS measurement error [4]-[8]:

– by adjusting the number of averaged signal samples N to ensure that the error component $sin(\omega N T_S)$ is equal to zero – see (4);

– by adjusting of the input signal initial phase to ensure that the error component $cos((N\text{-}1)\omega T_S + 2\alpha)$ is equal to zero; this is performed by adjusting the time of measurement start.

Both of these approaches require additional frequency measurements to calculate the number of averaged samples N or the value of the initial phase. To perform frequency measurement, one of the popular digital frequency measurement methods of a polyharmonic signal can be applied: the zero crossing technique; a method of determining the phase increment in time ("phasor" method), a quadrature demodulation technique and a spectral analysis method [13]-[14].

In addition to the listed approaches of RMS error reducing, there are other approaches:

– performing "sliding" RMS measurement technique and post-filtration of the RMS measurement results [7];

– performing averaging of the RMS measurement results obtained with a phase shift multiple of π [8].

The disadvantage of these approaches is a significant increase in the measurement time, which for a number of practical tasks is a significant disadvantage.

B. *The Case of Polyharmonic Signal*

For the case of RMS measurement of a polyharmonic signal, the analytical expression for the error estimation is a combination of (14) for all spectral components of the measured signal [4]-[8]:

$$\begin{aligned}\delta_{p0} \cong & -\sum_{i=1}^{M} \frac{X_i^2 \cos((N-1)\omega_i T_S + 2\alpha_i)}{2X_{ID}^2 N \sin(\omega_i T_S)\sin^{-1}(\omega_i N T_S)} + \\ & + \sum_{\substack{j,k=1 \\ j\neq k}}^{M} \frac{X_j X_k \cos((N-1)\omega_{j+k} T_S + \alpha_{i+k})}{2X_{ID}^2 N \sin(\omega_{j+k} T_S)\sin^{-1}(\omega_{j+k} N T_S)} - \\ & - \sum_{\substack{j,k=1 \\ j\neq k}}^{M} \frac{X_j X_k \cos((N-1)\omega_{j-k} T_S + \alpha_{i-k})}{2X_{ID}^2 N \sin(\omega_{j-k} T_S)\sin^{-1}(\omega_{j-k} N T_S)},\end{aligned} \tag{16}$$

where X_i denotes RMS of the i-th spectral component of the input signal; $\omega_i = 2\pi f_i = 2\pi i f_1$ denotes angular frequency of the i-th spectral component; α_i denotes initial phase of the i-th spectral component of the input signal; M denotes number of considered spectral components of the input signal; $\omega_{j-k} = \omega_j - \omega_k$; $\omega_{j+k} = \omega_j + \omega_k$; $\alpha_{j-k} = \alpha_j - \alpha_k$; $\alpha_{j+k} = \alpha_j + \alpha_k$.

The main approach of the error reduce, as in the case of a sinusoidal input signal, is associated with ensuring (4) by adjusting the samples number or by adjusting the sampling rate.

IV. THE RMS MEASUREMENT FOR THE CASE OF APPLICATION OF THE FIRST-ORDER APPROXIMATION POLYNOMYAL

A. The Case of Sinusoidal Signal

It can be seen that the measurement algorithm for the case of applying the approximation polynomials of the zero and first orders is slightly different – see (3) and (8). The resulting (14) can be applied to obtain an analytical expression for estimating the RMS measurement error for the case of first-order approximation polynomial. In the case of a sinusoidal input signal which is represented by:

$$x[n] = X\sin(\omega nT_S + \alpha)\,, \tag{17}$$

the RMS measurement error, taking into account (8) and (14), takes the following form:

$$\begin{aligned}\delta_{p1} \cong -\frac{\cos((N-1)\omega T_S + 2\alpha)\sin(\omega NT_S)}{2N\sin(\omega T_S)} + \\ +\frac{\sin((N-1)\omega T_S + 2\alpha)\sin(\omega NT_S)}{2N} = A_{p1}(\alpha,\omega,N).\end{aligned} \tag{18}$$

Fig. 1 shows the dependence of the RMS measurement error on the input sinusoidal signal frequency for the case of input signal initial phase equal to 0 radian (red dependence), π/6 radian (blue dependence) and π/3 radian (green dependence). The modeling parameters coincide with the parameters adopted for modeling the RMS error for the case of zero-order polynomial approximation. It can be seen from the figure that the initial phase for a case of first-order polynomial is practically not affected by the maximum error value and dependence behavior.

For the considered dependence, there are singular points for which the RMS measurement error is equal to zero. As can be seen from (17), the condition of equality of the error to zero is ensured when the factor $sin(\omega NT_S)$ is equal to zero, which is satisfied when the (4) is satisfied. Thus, in the case of a first-order polynomial, it is possible to provide a zero value of the RMS measurement error if the measurement time is chosen as a multiple of the input signal period (see (4)).

The dependence of the maximum value of the RMS measurement error when applying a first-order approximation polynomial on the input signal initial phase is shown in Fig. 2.

Fig. 3 shows the dependence of the RMS measurement error on the total measurement time. As in the case of a zero-order polynomial, it can be seen that with an increase of the measurement time, the error tends to decrease (in this case, the measurement time should correspond to (4) as close as possible).

Fig. 4 shows the dependence of the maximum RMS measurement error on the arbitrary initial phase of the input signal. It is seen that, as in the case of a zero-order polynomial, the dependence characterizes a singular points for which the RMS measurement error is equal to zero.

B. The Case of Polyharmonic Signal

In case of polyharmonic input signal:

$$x[n] = \sum_{i=1}^{M}\sqrt{2}X_i\sin(\omega_i nT_S + \alpha_i)\,, \tag{19}$$

signal $x^2[n]$ will contain the sum of harmonic components with frequencies $2\omega_k$, $(\omega_j - \omega_k)$ and $(\omega_j + \omega_k)$:

$$\begin{aligned}x^2[n] = \sum_{i=1}^{M}X_i^2(1-\cos(2\omega_i nT_S + \alpha_i)) + \\ +\sum_{\substack{j,k=1\\ j\neq k}}^{M}X_jX_k\big(\cos((\omega_j-\omega_k)nT_S + \alpha_j - \alpha_k) - \\ \cos((\omega_j+\omega_k)nT_S + \alpha_j + \alpha_k)\big).\end{aligned} \tag{20}$$

When performing integration of $x^2(t)$ according to (8), the total RMS measurement error is the sum of the RMS measurement error of individual spectral components, ((18) was obtained for the estimation of the this error). Then the resulting error takes the following form:

$$\begin{aligned}\delta_{p1} \cong -\sum_{i=1}^{M}\frac{X_i^2A_{p1}(\alpha_i,2\omega_i,N)}{2X_{ID}^2} + \\ +\sum_{\substack{j,k=1\\ j\neq k}}^{M}\frac{X_jX_k(A_{p1}(\alpha_{i+k},\omega_{j+k},N) - A_{p1}(\alpha_{i-k},\omega_{j-k},N))}{2X_{ID}^2}.\end{aligned} \tag{21}$$

where the value of polynomial $A_{p1}(\alpha, \omega, N)$ is determined according to (18), and used notation is represented in (16).

A decrease in the measurement error can be achieved by adjusting the averaged samples number in accordance with (4). This allows to reduce all the error components included in (21).

V. THE SECOND-ORDER APPROXIMATION POLYNOMYAL APPLICATION

A. The Case of Sinusoidal Signal

For the case of a sinusoidal signal (17), when applying the approximation by a second-order polynomial, the relative RMS measurement error can be represented as:

$$\begin{aligned}\delta_{p2} \cong -\frac{\cos(2\omega T_S)+2}{6\cdot(T+1)}\left(\cos\left(4\omega T_S\left(N+\frac{1}{2}\right)+2\alpha\right)+\right. \\ \left.+\frac{\sin(2\omega NT_S)\cos((N-1)\omega T_S + 2\alpha)}{\sin(2\omega T_S)}\right) = A_{p2}(\alpha,\omega,N).\end{aligned} \tag{22}$$

Expression (22) was obtained by substituting (16) into the RMS measurement algorithm represented by (12). The approximate character of expression (22) is caused by the application of the Taylor series expansion of the square root function (the expansion is up to two terms).

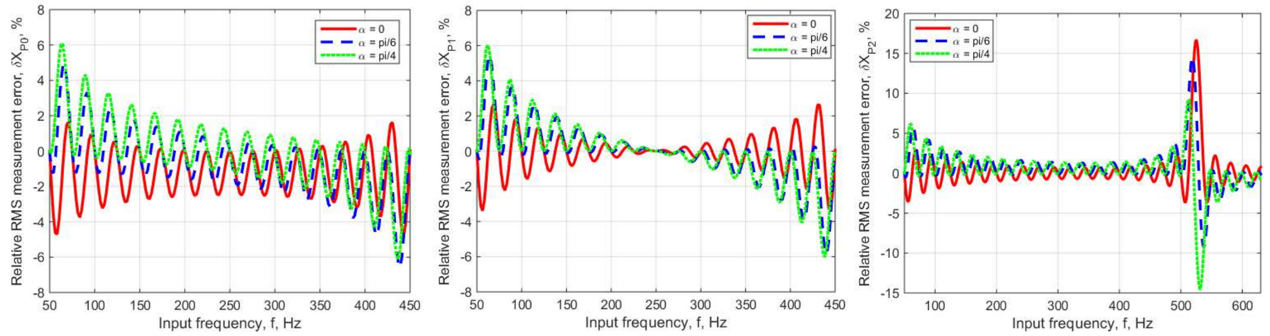

Figure 1. Dependence of the RMS measurement error on the input signal frequency for different values of the initial phase of the measured sinusoidal signal. From left to right: zero-order approximation polynomial, first-order polynomial, second-order polynomial.

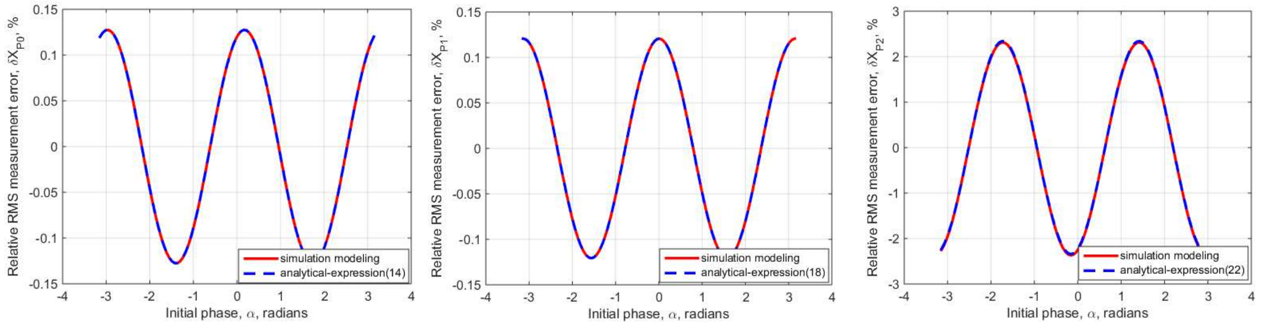

Figure 2. Dependence of the RMS measurement error on the initial phase of the measured sinusoidal signal. From left to right: zero order polynomial, first order polynomial, second order polynomial.

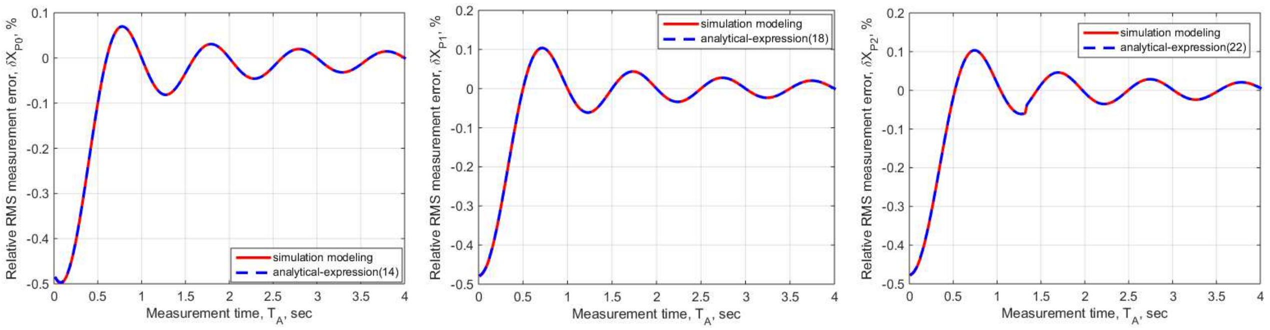

Figure 3. Dependence of the RMS measurement error of a sinusoidal signal on the total measurement time. From left to right: zero-order approximation polynomial, first-order polynomial, second-order polynomial.

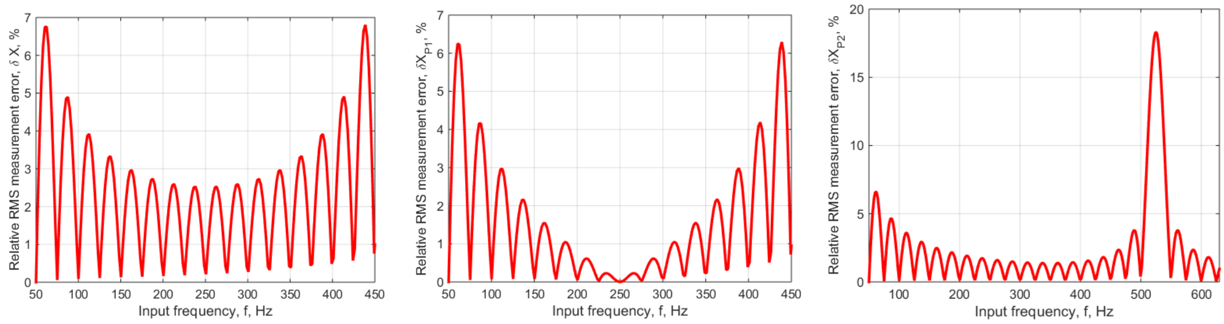

Figure 4. Dependence of the maximum value of the RMS measurement error on the frequency of the input sinusoidal signal. From left to right: zero order polynomial, first order polynomial, second order polynomial.

The dependence of the RMS relative measurement error for the case of applying a second-order polynomial on the input signal frequency (for three different values of the initial phase of the input signal) is shown in Fig. 1. It can be seen that the maximum value of the error weakly depends on the value of the initial phase (which corresponds to the case of applying a first-order polynomial). The frequencies ω_{opt}, at which the measurement error is equal to zero correspond to the following condition:

$$\frac{\sin(2\omega_{opt}NT_S)\cos((N-1)\omega_{opt}T_S+2\alpha)}{\sin(2\omega_{opt}T_S)}+ \\ +\cos\left(4\omega_{opt}T_S N+2\omega_{opt}T_S+2\alpha\right)=0. \quad (23)$$

The solution to this equation gives the following values of the "optimal" frequencies (ω_{opt}) for which the RMS measurement error is equal to zero:

$$\omega_{opt} = \frac{\pi k}{2(N+1)T_S}. \tag{24}$$

The resulting (24) corresponds to (4), which is valid for polynomials of the zero and first orders, taking into account the choice of the samples number (N parameter) and the sampling step for the case of applying the second order polynomial (13). Thus, the condition for the multiplicity of the measurement time to the true value of the input signal period to ensure the zero value of the error is equal to (4).

The dependence of the maximum value equal to the RMS measurement error when applying a second-order approximation polynomial on the initial phase of the measured signal is shown in Fig. 2.

Fig. 3 shows the dependence of the RMS measurement error on the measurement time. As in the case of polynomials of the zero and first orders, with an increase of the measurement time, the error tends to decrease (in addition the measurement time should correspond to (4) as close as possible). Fig. 4 shows the dependence of the RMS measurement error on the input signal initial phase. It is seen that, as in the case of the first order polynomial, there are singular points for which the RMS error is equal to zero.

B. The Case of Polyharmonic Signal

In the case of measurement of the polyharmonic input signal (19), the signal $x^2[n]$ can be represent in the form (20), that is, it is the sum of the spectral components with frequencies $2\omega_k$, $(\omega_j - \omega_k)$ and $(\omega_j + \omega_k)$: the integration error of which for the case of a second order polynomial approximation is determined by (22). Then the general expression for calculating the RMS measurement error of a polyharmonic signal is:

$$\delta_{p2} \cong -\sum_{i=1}^{M} \frac{X_i^2 A_{p2}(\alpha_i, 2\omega_i, N)}{2X_{ID}^2} + \\ + \sum_{\substack{j,k=1 \\ j\neq k}}^{M} X_j X_k \frac{A_{p2}(\alpha_{i+k}, \omega_{j+k}, N) - A_{p2}(\alpha_{i-k}, \omega_{j-k}, N)}{2X_{ID}^2}. \tag{23}$$

where polynomial values $A_{p2}(\alpha, \omega, N)$ can be calculated by (22), and the used designations are represented by (16).

VI. Application of Spline Functions

Along with the application of interpolation polynomials for performing recovery of discrete signals, spline functions are widely used [15]. The problem of using spline functions for the RMS measurement of sinusoidal and polyharmonic signals requires a large additional research.

In practice, the most popular is the so-called local spline (or Hermite spline) for which the behavior of the approximating function between samples is described by a third-order approximation polynomial function. In addition, for a given type of spline, the derivative of the approximating function and the original signal must coincide. These requirements underlie the principles for determining the spline coefficients (analytical expressions are presented in [15]-[16]). Regardless of the input waveform, 14 multiplication and 3 addition operations are required to perform the signal integration for one sampling step. In addition, the calculation of the spline coefficients requires 8 additional multiplication operations and the same number of addition operations. This significantly complicates the implementation of this method in real RMS measurement transducers.

From the point of view of the achieved accuracy, the application of splines allows to achieve a smaller maximum error of RMS measurement than all previously considered polynomial functions. However, as it was shown earlier, for polynomial functions there are "good" relationships (4) of the sampling frequency and signal frequency for which the RMS measurement error is equal to zero. The simulation results shows that for the case of application of spline function, these relationships are not met. The maximum value of the RMS measurement error that occurs when applying spline function can be estimated by using the following expression [15]-[16] (the expression is obtained for the case of a sinusoidal signal):

$$\delta_U = \frac{(2\pi)^4 \cdot f^4 \cdot T_S^4}{384 \cdot \sqrt{2}}. \tag{24}$$

where f denotes input signal frequency.

VII. Conclusion

As a result of the research performed, the following conclusions can be drawn:

– the possibilities of RMS measurement by the application of approximating polynomials of the zero, first and second order are considered;

– analytical expressions were obtained for estimating the RMS relative measurement error of a sinusoidal and polyharmonic signal (zero order polynomial – (14) and (16); first order polynomial – (18) and (21); second order polynomial – (22) and (23));

– a relation was obtained for the sampling frequency, signal frequency and the number of averaged samples, for which it is possible to achieve a zero value of the RMS measurement error by all the considered methods;

– it is shown that with an increase in the measurement time, the RMS relative measurement error by all the considered methods tends to decrease.

References

[1] Emanuel A.E. "Power definitions and the physical mechanism of power flow," Wiley Chichester, 2010.

[2] Emanuel A.E. "Powers in nonsinusoidal situations - a review of definitions and physical meaning," IEEE Transactions on Power Delivery, vol. 5, issue 3, pp. 1377–1389, 1990.

[3] “IEEE standard definitions for the measurement of electric power quantities under sinusoidal, balanced or unbalanced conditions,” IEEE Std. 1459–2010.

[4] Fan Wang; M. H. J. Bollen “Frequency-response characteristics and error estimation in RMS measurement,” IEEE Transactions on Power Delivery, vol. 19, issue: 4, pp. 1569-1578, 2004.

[5] Predrag B. Petrovic “Root-mean-square measurement of periodic, band-limited signals”, IEEE International Instrumentation and Measurement Technology Conference Proceedings, pp. 323-327, 2012.

[6] Andrey N. Serov; Nikolay A. Serov; Vadim A. Loginov; “Application of the Method based on Averaging of the Squares of Samples for the RMS Measurement of Polyharmonic Signals,” 2020 XXX International Scientific Symposium “Metrology and Metrology Assurance (MMA),” pp. 1-6, 2020.

[7] A.A. Kostina; P.M. Tzvetkov; A.N. Serov; "Investigation of the Method of RMS Measurement Based on Moving Averaging," 2020 55th International Scientific Conference on Information, Communication and Energy Systems and Technologies, ICEST 2020 - Proceedings, pp. 235–238, 2020.

[8] A.N. Serov; "An Approach of Reducing the RMS Measurement Error for the Measurement Method Based on Averaging of the Squares of Samples," Proceedings of the IEEE International Conference on Electrical Engineering and Photonics (EExPolytech), pp. 152-155, 2020.

[9] A. Serov; A. Shatokhin; A. Novitskiy; D. Westermann “Investigation of the method of RMS measuring based on the digital filtration of the square of samples,” 18th International Conference on Harmonics and Quality of Power (ICHQP), pp. 1-6, 2018.

[10] Ferrero A. “Harmonic power flow analysis for the measurement of the electric power quality,” IEEE transactions on instrumentation and measurement, vol. 60, issue 3, pp. 683-685, 1995.

[11] S. N. Mikhalin; V. M. Gevorkyan; “The problems of digital processing of signals in the system of automated power quality control and accounting quantity of electricity (ASQAE)”, MPEI Vestnik, No. 1, pp. 86-92, 2005.

[12] Andrey N. Serov; Alexander Novitskiy; Alexander A. Shatokhin; Steffen Schlegel; Ekaterina A. Dolgacheva; Dirk Westermann “The Influence of Power Frequency Deviation on the Active and Reactive Power Measurement Error with the Application of DFT,” 2019 20th International Symposium on Power Electronics (Ee), pp. 1-6, 2019.

[13] M. M. Begovic, P. M. Djuric, S. Dunlap and A. G. Phadke, “Frequency tracking in power networks in the presence of harmonics,” IEEE Transactions on Power Delivery, vol. 8, issue 2, pp. 480-486, 1993.

[14] B. Boashash; “Estimating and interpreting the instantaneous frequency of a signal. II. Algorithms and applications,” Proceedings of the IEEE, vol. 80, issue 4, pp. 540-568, 1992.

[15] Carl de Boor, “A Practical Guide to Splines,” Springer, New York, 1978, 348 p.

[16] Günther Nürnberger, “Approximation by Spline Functions,” Springer, New York, 1989, 244 p.