A New Nonlinear Dynamical Model with Three Quadratic Nonlinear Terms and Hidden Chaos

Sundarapandian Vaidyanathan, Esteban Tlelo-Cuautle, Aceng Sambas, Francesco Grasso

Abstract—A new nonlinear dynamical model with three quadratic nonlinear terms is introduced in this work. Lyapunov exponents are calculated to discover properties such as dissipativity and chaotic behavior. It is also established that there is no rest point for the proposed nonlinear plant signifying that the dynamical model undergoes hidden chaotic behavior. The proposed hidden chaotic system is implemented using electronic devices, which simulation results are in good agreement with MATLAB simulations.

Index Terms—Chaos, chaotic systems, hidden attractor, nonlinear plant, dynamical systems, circuit model.

I. INTRODUCTION

Nowadays, chaotic systems have a new classification that can be performed according to their nonlinear dynamics, so that one can distinguish two kinds of attractors: self-excited attractor and hidden attractor. In the former case, the attractor has a basin of attraction that is excited from unstable equilibrium point. As already mentioned in [1], the classical systems under this category are the classical nonlinear systems such as Lorenz's, Rössler's, Chen's, Lü's, or Sprott's systems. However, nowadays systems with hidden attractors have received great attention from both the theoretical and practical point of views. Self-excited attractors can be localized straightforwardly yet applying manual calculations. In contrast, one must develop specific computational procedures to identify a hidden attractor because the evaluation of their equilibrium points is difficult and they do not help in their localization.

Chaotic systems have a variety of engineering applications, as in the transmission of data. For example, Liu in [2] is proposing a direct acquisition algorithm using chaotic sequences to improve the communication systems based on chaotic DSSS signals, which aids in overcoming difficulties in different acquisition problems. Gohari et al. [3] proposed an algorithm for 3-D planning using chaotic maps for the motion planning and regulation of a quadrotor for boundary surveillance applications. For secure systems, Wang and Li [4] devised a color image encryption method which is constructed using Hopfield chaotic neural network. Hua et al. [5] dealt with the image encryption problem by proposing a cosine-transform-based chaotic system, which is useful in cryptography. Naser et al. [6] proposed a new approach to improve the security of multimedia systems using a chaotic map. Zarebnia et al. [7] investigated the image encryption problem for gray scale images with a numerical algorithm based on hybrid chaotic dynamical systems. Nasr et al. [8] investigated the problem of workspace coverage for mobile robotic motion with a flatness controller and multi-scroll chaotic attractor. Karakaya et al. [9] proposed a memristive chaotic circuit and discussed also FPGA implementation and TRBG based on it. Wang and Dong [10] discussed a 4-D autonomous quadratic hyperchaotic system from the classical Lorenz system and built an electronic circuit design. As one sees, the development of new chaotic systems opens the possibility of improving such kinds of applications.

This research work introduces a new nonlinear dynamical model with three nonlinear terms of quadratic polynomial type. To ensure that the proposed dynamical model is chaotic, Lyapunov exponents are calculated using time-series of the solutions, which also help to discover properties such as dissipativity and chaoticity. The dynamical analysis shows that there is no rest point for the proposed nonlinear plant, in this sense one can say that this new nonlinear dynamical model undergoes hidden chaotic behavior [1].

Section II introduces the new nonlinear dynamical model, which dynamical analysis confirms that the attractor is hidden. Numerical simulations are performed to observe the corresponding attractor in different phase-space portraits. Section III shows the scaling of the original new nonlinear dynamical model, in order to perform a circuit simulation using MultiSim. Finally, the conclusions are given in Section IV.
II. A NEW NONLINEAR HIDDEN CHAOS MODEL

The new nonlinear dynamical model with three quadratic nonlinear terms is given in (1). It can be appreciated that it has three state variables $\xi_1, \xi_2, \xi_3$.

\[
\begin{align*}
\dot{\xi}_1 &= \xi_2, \\
\dot{\xi}_2 &= a\xi_2\xi_3 + b\xi_2^2 - c\xi_2 - \xi_2\xi_3 \\
\dot{\xi}_3 &= \xi_2^2 - 1
\end{align*}
\]  

(1)

In the plant modeled by the dynamical equations from (1) one can see three coefficients $a, b$ and $c$. The appropriate values of these parameters to produce a positive Lyapunov exponent to confirm chaotic behavior are $(a, b, c) = (0.1, 0.1, 0.15)$. In this manner, the three Lyapunov exponents of (1) are estimated in MATLAB after $T = 1E5$ seconds for and $\xi(0) = (0.2, 0.2, 0.2)$. It results in $(LE_1, LE_2, LE_3) = (0.0536, 0, -0.1829)$. Therefore, in view of the signs $(+, 0, -)$ of the Lyapunov exponents and having a negative sum, it follows naturally that this new nonlinear dynamical model exhibits dissipative chaotic motion.

The equilibrium or rest points of the new chaotic model given by (1) are evaluated from the following equations:

\[
\begin{align*}
\xi_2 &= 0 \quad (2a) \\
a\xi_2\xi_3 + b\xi_2^2 - c\xi_2 - \xi_2\xi_3 &= 0 \quad (2b) \\
\xi_2^2 - 1 &= 0 \quad (2c)
\end{align*}
\]

From (2a), $\xi_2 = 0$, which contradicts (2c). In this manner, one concludes that there is not a defined rest point for this 3-D nonlinear dynamical model, and then it signifies that the model given by (1) exhibits hidden chaos motion [1].

The different phase-space portraits 2D views of this hidden chaotic attractor are shown in Figures 1, 2, and 3, and a 3D view is given in Fig. 4. The numerical simulations can be performed using the fourth order Runge-Kutta Method in MATLAB. In all cases the simulations were performed setting the parameters $(a, b, c) = (0.1, 0.1, 0.15)$ and using the initial conditions $\xi(0) = (0.2, 0.2, 0.2)$.

The mathematical model given in (1) can be implemented using embedded systems, as detailed in [11], or it can be implemented using analog circuits, as shown in the following Section.
III. MultiSim Circuit Model of the New Nonlinear Dynamical Plant with Hidden Chaos

The new nonlinear dynamical model given in (1), which dynamical analysis confirms that the attractor is hidden, can be simulated using commercially available electronic devices that can be modeled in the circuit simulator called MultiSim. In this manner, this Section shows the scaling of (1), as given by (3), and these equations are synthesized by multipliers, amplifiers and passive resistors and capacitors. It leads us to the circuit sketched in Fig. 5.

\[
\begin{align*}
\dot{\xi}_1 &= \xi_2 \\
\dot{\xi}_2 &= \frac{a\xi_2 \xi_1}{4} + \frac{b\xi_2^2}{2} - c\xi_2 - \frac{\xi_2^3}{4} \\
\dot{\xi}_3 &= \xi_2^2 - 4
\end{align*}
\]

(3)

Applying Kirchhoff’s current and voltage laws to Fig. 5 one gets (4), which associates the equations given in (3). Still \(\xi_1, \xi_2, \xi_3\) are the states variables but they are representing the voltages across the capacitors \(C_1, C_2\) and \(C_3\), respectively.

\[
\begin{align*}
\dot{z}_1 &= \frac{1}{R_1C_1} z_2 \\
\dot{z}_2 &= \frac{1}{R_2C_2} z_1 z_3 + \frac{1}{R_3C_2} z_2 - \frac{1}{R_4C_2} z_2 - \frac{1}{R_5C_2} z_2 z_3 \\
\dot{z}_3 &= \frac{1}{R_6C_3} z_2^2 - \frac{1}{R_7C_3} V_i
\end{align*}
\]

(4)

The values of the circuit elements in Fig. 5 are selected as: \(R_1 = R_6 = 400 \text{ k\Omega}\), \(R_2 = 16 \text{ M\Omega}\), \(R_3 = 8 \text{ M\Omega}\), \(R_4 = 2.7 \text{ M\Omega}\), \(R_5 = 1.6 \text{ M\Omega}\), \(R_7 = R_8 = R_9 = R_{10} = R_{11} = 100 \text{ k\Omega}\), \(V_i = 1 \text{ V}_{\text{DC}}\), \(C_1 = C_2 = C_3 = 1 \text{ nF}\). The multiplier and amplifier are commercially available, and the power supplies of all active devices are set to ±15 Volts.

The simulation results of the chaotic attractors using MultiSim are shown in Figures 6, 7 and 8, which are in good agreement with the simulation results shown in Figures 1, 2 and 3, respectively.
IV. CONCLUSION

We have shown the numerical and circuit simulation results of a new 3D dynamical model with hidden chaos. Lyapunov exponents were calculated to confirm that the system exhibits chaotic behavior, and the dynamical analysis showed the hidden attractor characteristic. The significant research contribution of this paper is the discovery of a new nonlinear dynamical model with three quadratic nonlinear terms exhibiting dissipativity and hidden chaoticity. The good agreement of the results between MATLAB and MultiSim confirmed the generation of hidden chaos.

REFERENCES


