Noise improvement for different BP filter designs

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Abstract - In this paper an analysis of noise through four different approaches to filter design is performed. The voltage noise spectral density and root mean square (RMS) value of noise voltage for the simply designed (SD) active RC, noise optimized (NO) active RC, operational transconductance amplifiers (OTA-C) and switched capacitor (SC) filter design are calculated for fourth-order Chebyshev band-pass filter, with central frequency f_c=4kHz, bandwidth 1kHz and pass band ripple a_{ripp}=0.1dB. For the achievement realization of all four designs a cascade of two second order sections are used. The analyses is performed using Matlab and SPICE (LTspice) programming tools.

Keywords - band-pass filter; noise; SC filter

I. INTRODUCTION

There are different ways to achieve the desired transfer function of the filter. The most commonly used realization is with the operational amplifiers (OA) which gives a stable (solid) voltage point output and also low impedance output. On the other hand, if high impedance output is desired, then realization with general transconductance amplifier (OTA-C) sections will be used. Such design gives us a constant current source on output.

Likewise, one of the realization of the same transfer function can be with switched capacitors (SC) where resistors are replaced with switched capacitors [1, 2]. This realization is good for design in monolithic integrated technology, where the filter is completely implemented in chip using thick and thin film technology.

Except satisfaction of elementary specifications on amplitude and phase filter characteristics, there exist additional parameters of filter quality. One of them is noise. The goal is to reduce noise as much as possible, because with superposition of noise on observed signal the information that the signal transmits can be covered. Through the sections of the paper will be demonstrating how to design filters on above suggested approaches with the desire to reduce noise in filter.

II. REALIZATION

The normed transfer functions of the observed Chebyshev band-pass filter are given by:

\[ H_1(s) = \frac{0.4525s}{s^2 + 0.3475s + 1.415}, \quad (1a) \]

\[ H_2(s) = \frac{0.4525s}{s^2 + 0.2048s + 2}, \quad (1b) \]

\[ H(s) = \frac{0.4525s}{s^2 + 0.3475s + 1.415}, \quad (1c) \]

\[ H_1(s) = \frac{-5.3542 \times 10^3}{s^2 + 4.1764 \times 10^3 s + 7.5469 \times 10^8}, \quad (2a) \]

\[ H_2(s) = \frac{-6.3969 \times 10^3}{s^2 + 3.4956 \times 10^3 s + 5.2869 \times 10^8}, \quad (2b) \]

\[ H(s) = \frac{-6.3969 \times 10^3}{s^2 + 3.4956 \times 10^3 s + 5.2869 \times 10^8}, \quad (2c) \]

\[ H_1(s) = \frac{3.4251 \times 10^7 s^2}{s^4 + 7.672 \times 10^3 s^3 + 1.2979 \times 10^9 s^2 + 4.8462 \times 10^12 s + 3.99 \times 10^{17}} \quad (2d) \]

A. Continuous-time active-RC filters

In Fig. 1 the Single Amplifier Biquad (SAB) topology of the band-pass filter is given.

Realization is performed as a cascade of two second order sections. The circuit transfer function is given as:

\[ H(s) = \Pi_{i=1}^{4} H_i(s), \quad (3a) \]

Figure 1. Fourth-order OA band-pass filter
Comparison of the equations (4a, 4b) with (2a-2c) element values of OTA-C filter was calculated and shown in Table 2. Frequency responses are shown in Fig. 4.

**Figure 4.** Frequency response of the Fourth-order OTA-C BP filter
C. Discrete time SC filter

The discrete time SC filter based on Sallen-Key (SAK) topology is shown in Fig. 5.

The forward Euler transformation \( s \rightarrow \frac{(z-1)}{T_s} \) was applied for calculation of the discrete time transfer function. Where \( T_s \ (f_s=4900f_c) \) is a sampling time and which has been calculated as \( T_s=1/f_s \).

\[
H(z) = H_1(z) \cdot H_2(z), \tag{5a}
\]

\[
H_1(z) = \frac{5.8023 \cdot 10^{-4} \cdot (z-1)}{z^2 - 1.9997z + 0.9997}, \tag{5b}
\]

\[
H_2(z) = \frac{5.8023 \cdot 10^{-4} \cdot (z-1)}{z^2 - 1.9996z + 0.9996}, \tag{5c}
\]

\[
H(z) = \frac{3.3667 \cdot 10^{-7} \left(z^2 - 2z + 1\right)}{z^4 - 3.9992z^3 + 5.9977z^2 - 3.9977z + 0.9992}. \tag{5d}
\]

The transfer function of discrete time SC filter based on Sallen-Key topology is given by:

\[
H(z) = \prod_{i=1,2} \left. \frac{a_{2i}}{a_{4i}} \cdot (z-1) \right. \cdot \frac{a_{1i}}{2i} \cdot z^2 - a_{3i} \cdot z + a_{4i}, \tag{6a}
\]

\[
G_i = 1 + \frac{C_{4i}}{C_{5i}}, \tag{6b}
\]

\[
a_{2i} = \frac{C_{1i} \cdot C_{2i} \cdot C_{5i}}{C_{4i} \cdot C_{2i} \cdot R_{li}}, \tag{6c}
\]

\[
a_{3i} = 2C_{1i}^2 C_{2i} + C_{1i} C_{2i} C_{5i} + C_{2i}^2 C_{2i} R_{li} + C_{2i} C_{2i} C_{2i} R_{li} + C_{2i} C_{2i} C_{2i} R_{li} + C_{2i} C_{2i} C_{2i} R_{li}, \tag{6d}
\]

\[
a_{4i} = C_{2i}^2 C_{2i} + G_i C_{2i} C_{2i} C_{2i} R_{fi}. \tag{6f}
\]

Comparing the equations (5a-5c) with (6a-6f) element values were calculated and presented in Table 3. From Fig. 6 and Fig. 2, it can be seen that the Z-transformation hasn't got undesired effects on filter specifications.

![Figure 5. Fourth-order SC band-pass filter (SAK topology)](image)

![Figure 6. Frequency response of the Fourth-order SC band-pass filter](image)

<table>
<thead>
<tr>
<th>TABLE III. ELEMENT VALUES OF SC FILTER</th>
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<td>( 1^\text{st} ) section</td>
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III. NOISE

A. Noise in filter with OA

Fig 7. shows an equivalent circuit of the active-RC filter for the calculation of the voltage noise spectral density. Parallel to the resistors have been added current noise sources: \( I_{ni} = \sqrt{4kT/R} \), whilst to the voltage noise sources \( E_n = 2.5 \text{nV/Hz} \) for LT1007 have been added at the input of the operational amplifiers. The input current noise source of operational amplifiers were neglected, because its values are much smaller than voltage noise sources, for instance \( I_e=0.4 \text{pA/Hz} \).

The voltage noise spectral density is calculated by the given equation [4]:

\[
V_n^2(\omega) = \sum_{i,j} T^2_{V,i}(j\omega) \left( E_{n,i} \right)^2 + \sum_{j} T^2_{V,j}(j\omega) \left( E_{n,j} \right)^2.
\]  

(7)

In this equation \( T_{V,i} \) and \( T_{V,i} \) represent the transfer functions of the voltage noise sources \( E_{n,i} \) and the current noise sources \( I_{n,i} \) respectively. Fig. 8 (a) shows the total voltage noise spectral density while Fig. 8 (b, c) shows noise components for both sections. The differences between simulated and calculated values can be considered as a result of using theoretical values based on thermal noise sources in calculations and real elements noise models using SPICE. The RMS value has been calculated by the equation:

\[
\left( E_n^2 \right)^{1/2} = \int_{\omega_1}^{\omega_2} V_n^2(\omega)d\omega.
\]

(8)

In the desire to reduce voltage noise spectral density of filter we used the optimization algorithm given in [3]. All element values are taken from [3], recalculated and tested in Matlab and SPICE in order to give us smaller noise results. The total voltage noise spectral density of NO filter design is shown in Fig. 9 (a). Noise components for both sections are given through the Fig. 9 (b, c).

Comparing the noise RMS values of filters, we can conclude that optimized filter gave smaller noise RMS value.

B. Noise in OTA-C filter

The equivalent circuit for the calculation of the voltage noise spectral density of the observed OTA-C filter is shown in Fig. 10.

Voltage noise spectral density (7) is calculated. The total voltage noise spectral density components are given in Fig. 11 (a). The differences between simulated and calculated values can be considered as a result of using theoretical values based on thermal noise sources in calculations using MATLAB and real elements noise models using SPICE.
LT1228 was used as a transconductance amplifier in this design. The input voltage noise source of this

Figure 10. Equivalent electrical noise circuit of the Fourth-order OTA-C band-pass filter

Figure 11. Noise voltage spectral density of Fourth-order OTA-C band-pass filter and its components. (a) Total noise voltage spectral density: simulated (dashed) and calculated (full). Noise RMS value is $E_n=2.4384 \mu V$. (b) Section 1 noise voltage spectral density components. Noise RMS value is $E_n=1.5715 \mu V$. (c) Section 2 noise voltage spectral density components. Noise RMS value is $E_n=1.8645 \mu V$. 

Figure 9. Noise voltage spectral density of Fourth-order NO band-pass filter and its components. (a) Total noise voltage spectral density: simulated (dashed) and calculated (full). Noise RMS value is $E_n=20.412\mu V$. (b) Section 1 noise voltage spectral density components. Noise RMS value is $E_n=12.422\mu V$. (c) Section 2 noise voltage spectral density components. Noise RMS value is $E_n=16.197\mu V$. 

Fig. 11 (b, c) shows noise components for both sections.
transconductance amplifier is $E_n=6 \text{ nV/} \sqrt{\text{Hz}}$ and the input current noise source is $I_n=1.4 \text{ pA}/ \sqrt{\text{Hz}}$. The current noise source is much smaller and because of that it was neglected in calculation procedure.

C. Noise in SC filter

The fourth-order SC band-pass filter is shown in Fig. 5. The equivalent circuit for the calculation of voltage noise spectral density is based on [5]. Fig. 12 shows total voltage noise spectral density and its components.

D. Comparison

If the RMS voltage noise value is measured between 0.1fc to 10fc as the indicator of noise reduction, comparison of noise in all presented designs is given in Table 4. Fig. 13 shows voltage noise spectral density for all filter designs which are discussed in this paper.

IV. Conclusion

In this paper, noise improvement using different filter designs is analyzed. Analyses of four different designs are done. As noise measure RMS voltage noise value is calculated between 400 Hz and 40 kHz. Noise analysis is done for SD and for NO design filters based on operational amplifiers. Noise improvement for NO is 2.5 times. Noise analyses for OTA-C and SC designs are also done. They gave improvement of more than 20 times for OTA-C design in comparison with SD and 10 times for SC design in comparison with SD. In the end, considering RMS voltage noise as a numeric indicator, it is shown that the OTA-C filter design gives the best noise reduction.

REFERENCES


