Optimisation of factory floor layout using force-directed graph drawing algorithm

T. Kanduč* and B. Rodič*
* Faculty of Information Studies, Novo mesto, Slovenia
tadej.kanduc@fis.unm.si

Abstract - In this paper we tackle the problem of factory floor layout optimisation in a furniture factory. The goal of optimisation was the reduction of manufacturing costs, specifically reduction of the costs of product transport between machines by reducing the product travel distance and consequently the amount of labour. Limiting factors are the costs of relocating the machines to the optimal positions. We have analysed the current state of the manufacturing system by developing a detailed discrete event simulation (DES) model. The DES model was then used to develop optimisation methods and verify optimisation scenarios. Finally, from output results of the simulation we developed a novel heuristic method, based on force-directed graph drawing algorithms which has produced significantly improved floor layouts.

I. INTRODUCTION

Analysis and optimisation of complex manufacturing systems can be a demanding and time-consuming work since the processes are interwoven and impossible to analyse separately. Processes are usually also too complex to be modelled and optimised with exact mathematical methods. Methods more suited for modelling complex manufacturing systems include discrete event simulation (DES) modelling, which despite its ease of use can provide enough details to understand and analyse the logistic processes in a manufacturing system.

Construction of a DES simulation model requires that the data that describe the manufacturing processes are obtained, extracted and analysed in structured form. Integration of simulation software, auxiliary applications and databases with the model is necessary in order to have one coherent system that operates accurately despite changes in manufacturing processes or variations of scenarios in work orders.

System or process optimisation can be performed in two ways: by changing model parameters, or by changing the model structure. Changing the parameters is often implemented by built-in optimisation methods within the simulation tools, while modifications of model structure are not. In our case the optimisation involved changing the factory layout, which was modelled by a network model, which is part of structure of the model.

Optimisation through modification of model structure can be performed by constructing several versions of the model and input data (i.e. scenarios) and comparing simulation results. To accelerate the development of model versions and scenarios one can construct algorithms that build or modify simulation models according to model input data. This is especially useful in cases of large simulation models and if the model versions are prepared by an optimisation algorithm. Automated model building and modification however requires that the model structure can be modified with an algorithm, without manual interventions. To this end we have developed an automated model construction algorithm, which will be extensively described in our further publications. The focus of this paper is on the novel heuristic optimisation method.

In the paper we present the novel factory layout optimisation method, based on force-directed graph drawing. The method was developed in the course of manufacturing process optimisation project in a furniture company. Our goal was to investigate how the layout of machines on the factory floor affects the efficiency and costs of manufacturing processes. Furthermore, the objective was to develop an optimised layout that would be implemented in real life. In [1] we focused on optimising the total distance the manufactured products need to travel on the floor. In this paper we extend the problem by minimising two dependant criteria. The considered criteria are the total costs of one-time machine relocation and the labour costs in transport of products between the machines as a result of changes in machine relocation.

A. Problem situation

The factory floor contains approximately 140 machines of different types – from simple carpeting workstations to expensive multi-purpose CNC machines. Company catalogue contains more than 30,000 different products and semi-finished products. Every item is manufactured according to the prescribed bill of materials (BOM) and its technical procedure. In BOM, required semi-finished products and materials to manufacture the given product are listed. The technical procedure data describe the sequence of operations in the manufacturing of the given product. The number of operations per finished product ranges from 3 to 20 with the average of 8 operations. The technical procedure data include a group of equivalent machines (GEM) for each operation and standard machine setup and machine operation duration times. The times are listed as single numbers, not as distributions. Compound products are manufactured by joining smaller semi-finished products also produced in the factory.
Production scheduling is based on customer orders and their priorities and performed using the Preactor scheduling system. Our simulation model uses schedule generated by Preactor or prepared by factory planners, without modification. Typically, there are several open customer orders in active production schedule. Products are manufactured in series ranging from approximately ten to several hundred pieces with a typical series containing approximately 30 pieces. For every operation, unfinished products are stored at input pallets or carts next to the machine. Machined products are stored on the output palette or carts until the entire series of products has completed the current operation. The series is then moved to the next location (machine) manually. Therefore the total number of required transport workers depends on the number of simultaneous carts in transition between machines. The assumption in the optimisation process was that shorter transport routes will result in shorter transport worker tasks, increasing the worker availability and potentially reducing the required number of transport workers.

The Preactor database defines GEM for each operation from the technical procedures. Most operations can be performed on several machines, either because there are several identical machines available, or because the operation can be performed on different multi-purpose machines. During the manufacture, a machine is dynamically selected from the GEM for the entire series of a product, according to machine preference and machine availability. Typically a machine that is currently least loaded within the GEM is selected. Hence the manufacturing process can be referred to as “flexible manufacturing”. The simulation model has to reflect this flexibility and model the machine groups, group selection and machine selection process for each technical procedure (i.e. each product).

The manufacturing process involves a large set of different products and monthly variations of open orders. Therefore, developing a static simulation model that would cover all available (i.e. 30,000) products and their technical procedures is not realistic. Instead, the simulation model is built automatically from a model template by an algorithm that uses the database of technical procedures and the database of currently open orders to place only the necessary machines in the model. The model reads the technical procedures and BOMs dynamically during the simulation to adjust machine model parameters and assembly of products.

B. Optimisation goal

The costs of manufacturing can be reduced by decreasing the need for labour in the transport of products between machines through better machine placement, i.e. factory layout. However, relocation of a machine is a difficult and costly measure and disrupts the manufacturing process. Therefore it makes economic sense to move a machine only if the relocation will considerably reduce product travel distance and consequently the need for labour. Namely, relocating the machines is associated with additional expenses: moving the machine costs $g_i$ amount of currency. Presumably, good candidates for relocation are machines with high product flows.

Various model statistics are measured during the simulation runs. Most important in this part of the project is the flow of products $f_{ij}$ between a pair of machines $m_i, m_j, i = 1, 2, ..., N$. Flow $f_{ij}$ represents the total amount of volume of products that was directly transported between these two machines. From the product flow $f_{ij}$ we straightforwardly compute cost flow $f_{ij}$, i.e., cost to move all products between the two machines for distance of 1 m. The distances between the machines are yet not known and are obtained from the optimisation of the factory floor layout. To reduce the overall costs we need to solve the following optimisation problem

$$\min_{\{p_1, p_2, ..., p_N\}} \left( \sum_{i=1}^{N} f_{ij} \cdot d(p_i, p_j) + \sum_{i=1}^{N} g_i \right), \quad (1)$$

where $p_i$ represents position of machine $m_i$ and $d$ is a distance functional. If we neglect the costs $g_i$ and restrict the positions $p_i$ to a predefined grid, the problem simplifies to well-known quadratic assignment problem (QAP). The latter is NP-hard optimisation problem and exact optimisation methods are successful only for smaller number of machines, usually around $N < 30$. In our case, where $N = 140$, the exact methods are not feasible and we need to apply heuristic methods instead, which in practice return a near optimal solution.

C. Previous research (review of literature)

Simulation is commonly used for the evaluation of scenarios [2], [3], [4]. However, the models developed with the visual interactive modelling method (VIM) are usually manually constructed through careful analysis of the real-life system and communication with process owners. Automated model development is more common with methods that allow easier and more standardized formal description of models, e.g. Petri nets [5], [6]. Automation of model construction and adaptation can importantly facilitate the development of models of complex systems [7], [8] and generation of simulation scenarios.

Several papers deal with factory layout optimisation, with paper [9] stating that multiproduct enterprises requires a new generation of factory layouts that are flexible, modular, and easy to reconfigure. Evolutionary optimisation methods are often proposed due to problem complexity [10]. Layout optimisation problem is identified as hard combinatorial optimisation problem and the simulated annealing meta-heuristic resolution approach is proposed to solve to problem [11]. A novel particle swarm optimization method is proposed by [12] for intelligent design of an unconstrained layout in flexible manufacturing systems.

Factory layout design optimisation is further discussed in [13], [14], [15]. Authors [13] propose a new facility layout design model to optimise material handling costs. Sources [14] and [15] propose genetic algorithm based solutions to respond to the changes in product design, mix and volume in a continuously evolving work environment.

The layout optimisation problem is similar to well-known quadratic assignment problem (QAP) [16], [17]. But the problem in its original form does not consider including fixed expenses to move the facilities (machines).
to new locations as it is in our case. Hence QAP algorithms in the original form cannot be used to solve our optimisation problem.

There are numerous methods for graph drawing and class of force-directed methods are one of the most commonly used in practice due to their simplicity and visually appealing representation of the graphs (see [18], [19], [20] and references therein). One of the earliest and still commonly used method is from the authors Fruchterman and Reingold [20].

II. METHODOLOGY

We have used Discrete Event Simulation methodology to develop a simulation model that captures all of the important features of manufacturing processes. The purpose of the model is verification of new manually or algorithmically generated floor layouts. Optimisation of floor layout is conducted in cooperation with experienced manufacturing planners, managers and other experts within the company, and is greatly facilitated by our novel heuristic optimisation method that is employed to generate new layout scenarios, i.e. to search for the optimal layout within a large set of possible layouts.

A. Machine model

Every machine is modelled as a machine block in Anylogic as shown in Fig. 1. On the input of the block, carts filled with products enter the system. Products are sorted according to their type at cartSource. The corresponding sinks, sink1 and sink2, monitor products on input pallets. Once the product is chosen for operation, it is injected at source. Blocks setUpMachine and machineDelay are standard service blocks. Block waitForWholeSeries plays a role of output pallet. Products wait there until complete series of products is finished. Some products need to wait at dryingDelay according to the technical procedure (paint, varnish drying, etc.). Filled carts are injected at cartSource and moved to the next location at moveCartTo. Output of the main machine block sends a cart to the input of the next machine.

B. Optimisation problem

In this section we describe the problem of finding the factory floor that minimises the overall costs of machine relocations and costs of workers that move carts between the machines. The problem is to find the optimal mathematical network, in which nodes of the network represent the machines on the factory floor. Weighted edges between the nodes represent transactions between the machines. Additional edges connect the sought network with the fixed old machine positions. Real routes on the floor between the machines are neglected in this case, since it considerably complicates the optimisation problem. The optimisation method should only propose a basic outline of the layout, since the final layout needs to be further tuned by the company experts to meet other less precise criteria.

Factory floor is described as a region $\Omega$ in the plane $\mathbb{R}^2$.

We will simplify the problem by restricting $\Omega$ to the rectangular shape,

$$\Omega = \{(x, y) \in \mathbb{R}^2; x_{\min} \leq x \leq x_{\max}, y_{\min} \leq y \leq y_{\max}\},$$

where $x_{\min}, x_{\max}, y_{\min}, y_{\max}$ represent boundaries of the rectangular factory floor.

Let us denote machines by $m_i$, $i = 1, 2, ..., N$. Position of the machine $m_i$ is described by

$$p_i = (x_i, y_i) \in \mathbb{R}^2.$$

Each machine takes certain amount of space which can be conveniently described by a metric rectangular-like ball $B_{r_i}(p_i)$ with radius $r_i$ and centre $p_i$ in $\infty$-norm $L_\infty$,

$$B_{r_i}(p_i) = \{(x, y) \in \mathbb{R}^2; d_\infty((x, y), p_i)) = \max\{|x-x_i|, |y-y_i|\} < r_i\}.$$  

Let $p_i'$ be old position (before relocating) of machine $p_i$. For every pair of machines $m_i$ and $m_j$, $i, j = 1, 2, ..., N$, we obtain overall costs $f_{ij} \geq 0$ to move the products between the machines for 1 m. The quantity is obtained as a result of the simulation of the manufacturing processes. The cost is proportional to the amount of products that is transferred between the machines. The cost is also proportional to the considered observed time interval. In practice due to high costs of relocating the machine, the observed time interval for workers’ costs should be at least a year.

Manhattan distance $d_{hi}$ between two points $p_i, p_j$ is defined as
Manhattan distance is a good approximation of actual route distance between the machines on the factory floor.

Relocation costs for machine \( m_i \) from position \( p_i^j \) to \( p_i^j \) is defined as a value \( g_i > 0 \) if \( d_M(p_i^j, p_i^j) \neq 0 \) and zero cost otherwise. Costs \( g_i \) were obtained from company planners who consider several criteria such as switching on and off the machine, construction work, resupplying costs, cleaning, etc.

The optimisation problem of minimising the total cost is described as

\[
\min_{(p_1, p_2, \ldots, p_N)} \left( \sum_{i,j=1}^{N} f_{ij} \cdot d_M(p_i, p_j) + \sum_{i=1}^{N} g_i(p_i^j, p_i^j) \right),
\]

where positions \( p_i \) must satisfy the conditions

\[
B_{ri}(p_i) \cap B_{rj}(p_j) = \emptyset
\]

for every \( i \neq j \) and

\[
B_{ri}(p_i) \subseteq \Omega
\]

for every \( i = 1, 2, \ldots, N \).

The first conditions states that the regions of machines must not intersect between each other and the second that every machines must lie entirely on the factory floor.

### III. RESULTS

The main result of our project described in this paper is the novel force-directed graph drawing optimisation heuristic method that generates new factory floor layouts by minimising overall costs of repositioning the machines and costs of workers that move the carts between the machines. Costs for machine relocation were estimated by the company planners. Workers’ costs depend on three parameters: manufacturing process, observed time interval and distances between the machines. For predefined manufacturing processes we obtain flows of products between the machines. From this data we can directly obtain workers’ costs \( f_{ij} \) to move the products for 1 m. Taking into account \( f_{ij} \), distances between the machines \( d_M(p_i, p_j) \) and relocation costs \( g_i \) we obtain the overall transport costs of the manufacturing processes.

#### 1) Force-directed graph drawing algorithm

The heuristic optimisation algorithm for assigning positions \( p_i \) to machines \( m_i \) is based on force-directed graph drawing methods. Initial position of the machines is chosen randomly but machines with greater product traffic are added to the system earlier. The machines are positioned according to the applied forces in the system and converge to configuration with a local minimum of the overall energy. After the systems converges or the maximum number of iterations is reached, the simulation is finished.

The optimisation was implemented in the system dynamics (SD) methodology in Anylogic software. The method is an extension of the original method that was used recently in QAP of minimising the total distance of machines [1].

Every machine is presented as a node on a plane. To every node \( n_i \) we prescribe the corresponding repelling force \( F_{ij} \) to all other nodes \( n_j \),

\[
F_{ij} = H_{ij}(||p_j - p_i||_\infty) \cdot \frac{p_j - p_i}{||p_j - p_i||_\infty},
\]

where \( H_{ij} \) is a positive monotonically decreasing function. For our algorithm we used

\[
H_{ij}(r) = \begin{cases} c_i \cdot (r_i - r)^2, & r \leq r_i \\ 0, & r > r_i \end{cases},
\]

where \( c_i \) and \( r_i \) are distance influence parameters. Repulsive forces keep the nodes away from each other since we want sufficient space between the machines.

For every pair of nodes \( n_i, n_j \) we define a weighted edge \( e_{ij} \) with weight \( f_{ij} \), i.e., cost to move all products between \( m_i \) and \( m_j \) for 1 m. Attractive forces between the nodes are defined as

\[
G_{ij} = -f_{ij} \cdot I_{ij}(||p_j - p_i||_1) \cdot \frac{p_j - p_i}{||p_j - p_i||_1},
\]

where \( I_{ij} \) is a positive monotonically increasing function. In our case, \( I_{ij} \) is defined as \( I_{ij}(r) = d_M(p_i, p_j) \). Attractive forces move the nodes with large edge weights closer to each other.

Attractive force representing machine \( m_i \) relocation cost is defined as

\[
J_i = \begin{cases} \{ g_i \cdot d_M(p_i, p_i'), \ d_M(p_i, p_i') < 5 \\ g_i, \ d_M(p_i, p_i') \geq 5 \} \end{cases}
\]

Note that \( J_i \) needs to be a continuous function of the distance, otherwise the optimisation method would not converge to a stable node configuration. Distance of 5 m is a threshold at which moving the machine is still considered.

To keep the nodes inside the prescribed location \( \Omega \), we also need to define forces that pull the nodes back to the interior if they are outside the prescribed region \( \Omega \),

\[
K_i = \begin{cases} 0, & p_i \notin \Omega \\ \text{dist}(\cdot, \Omega), & p_i \in \Omega \end{cases}
\]

and \( \text{dist}(\cdot, \cdot) \) is a function measuring the distance between objects.

Screenshot of the running algorithm implemented in Anylogic is shown in Fig. Machines (nodes) are presented as circles on the rectangular factory floor.
For our furniture factory floor layout we tested the algorithm for two observed time intervals: one year and three years. The distances between the machines were optimised by the algorithm to obtain the lowest overall costs. The results of optimisation are presented in Table 1. For both observed time intervals we reduced overall costs by decreasing the number of moved machines: for one year by 12 and for three years by 32 respectively. The reduction considers the costs of machine relocation, i.e. only the most relevant machines are moved.

Further important results of the project are the automated model construction algorithm and the discrete event simulation model in Anylogic. The model is integrated with a manufacturing scheduling system database and is used for in-depth analysis of the manufacturing process and verification of optimisation scenarios.

IV. CONCLUSION

In this phase of the project the deliverables include the novel force-directed graph drawing optimisation heuristic method, the automated model construction algorithm and the integrated detailed simulation model in Anylogic that communicates with external database files. The model captures all important features of the manufacturing process and servers as an indispensable tool for in-depth analysis and extracting additional data for further optimisation.

The described force-directed graph drawing optimisation heuristic method has been used to generate several new factory layouts which have been validated by the factory planners. The new layouts were significantly better than the current layout and manually produced layouts prepared by the factory planners. The optimisation method allows the company to significantly reduce the product transport costs by decreasing the number of workers performing in-factory transport of products. The factory layouts will be further refined and implemented in the course of ongoing micro-logistic optimisation project. During this project we will further develop the factory model by modelling scenarios, including automatisation of machine servicing using robots and new automated transport system that uses automated guided vehicles (AGVs).

### TABLE I. OVERALL COSTS AND NUMBER OF MOVED MACHINES FOR DIFFERENT FACTORY FLOOR LAYOUTS

<table>
<thead>
<tr>
<th>Time interval</th>
<th>Layout</th>
<th>Cost</th>
<th>Number of moved machines</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>current</td>
<td>50,800 €</td>
<td>0</td>
</tr>
<tr>
<td>1 year</td>
<td>optimal</td>
<td>47,400 €</td>
<td>12</td>
</tr>
<tr>
<td>3 years</td>
<td>current</td>
<td>149,400 €</td>
<td>0</td>
</tr>
<tr>
<td>3 years</td>
<td>optimal</td>
<td>139,700 €</td>
<td>32</td>
</tr>
</tbody>
</table>

REFERENCES


