The Analysis of Geometry, Coverage and Capacity of Heterogeneous Networks

Jasmin Musovic*, Nina Slamnik†, Mesud Hadzialic†, Faris Tankovic‡ and Irfan Krijestorac†
*Communications Regulatory Agency
† Faculty of Electrical Engineering, University of Sarajevo
‡ BS Telecom Solutions d.o.o.
Sarajevo, Bosnia and Herzegovina
Email: jmusovic@rak.ba, nslamnik1@etf.unsa.ba, mhadzialic@etf.unsa.ba, ftankovic1@gmail.com, ikrijestor1@etf.unsa.ba

***Abstract*—Mobile operators are confronted with the fact that a large portion of the calls and data transfer take place indoors. The aim of this paper is to present an analytical model for the planning and use of network resources in the heterogeneous networks (multitier, multistandard) whose topology will enable network enhancements of indoor and outdoor coverage, capacities and reliability of communication, reduction of investment costs and maintenance of the network, as well as reduction of electricity consumption. This model should set the theoretical framework in which it is possible to upgrade efficiently the existing network with the least costs of network deployment, while at the same time providing the capacity to meet user requirements in the necessary places and at the required time. Accordingly, a set of values of network resource parameters is presented as one of the main results achieved by this model. Hence, this set grasps parameters' values which enable increased energy and spectral efficiency, as well as required capacities, the satisfactory level of quality of service, and rational energy consumption within the real network in the time domain. In addition, the results include the establishment of a connection with the necessary limit values of the radio signal parameters at the cell boundaries, considering all cell tiers.**

Keywords: Heterogeneous cellular networks, Poisson point processes, spectral efficiency, deployment efficiency, HetNet, HNC, CapEx, OpEx.

I. Introduction

Fulfilling the users' needs for increased data traffic is achieved by optimizing the use of radio spectrum and by deploying low-range cells layered across existing macrocellular network architecture in tiers. In this paper we consider multiple access network design in a certain area on a downlink only. The basic metrics we will focus on are signal coverage and capacity.

- *Coverage* - The so-called service zone is an area in which user terminals can correctly receive both the control signaling and the data traffic at some minimum rate from the base station (BS). This means that the received signal from the BS is strong enough so the received signal power from the BS exceeds some threshold, and that the received signal to interference plus noise ratio (SINR) at the user exceeds some minimum value. SINR is the key indicator of coverage. However, for the sake of deployment efficiency analysis and though the modern cellular networks such as LTE and LTE-A enabled receivers with very low threshold on received signal power within a large distance from the transmitting BS, we are analyzing the signal power together with SINR.
- *Capacity* - It is known that there is a relation between the available data link rates and SINR on that link. This means that the link capacity is also dependent on SINR at user reception terminals [1].

A. Structure of the Problem

1) SINR calculation problem: Since coverage and capacity are both related to the SINR at the users receiver, we shall focus on the statistical properties of this SINR [2]. Three main deployment parameters for a network which will satisfy user demands in coverage and/or capacity are as follows:

- the number of tiers of the network,
- the densities of the BSs in the tiers, and
- the transmit powers of the BSs in the tiers.

2) Deployment Efficiency: Each service provider will try to optimize the costs of building and upgrading the network, i.e. Capital Expenditures (CapEx) and Operating Costs (OpEx) while meeting the requirements for coverage and/or capacity. In other words, in addition to the complex criteria of signal coverage and capacity requirements, there is a complicated utility function that is dependent on economic variables such as the cost pricing model, CapEx, OpEx and expected revenue. In this paper, these problems will be analyzed by performing the relationship between cost parameters and the distribution of SINR within the area of a heterogeneous network. Based on the selected SINR distribution, the service provider can select a set of base station allocation parameters based on its useful function, which considers economic reasons.

3) Candidate serving BSs and the serving BS: At an arbitrary temporal point t_0 network resources consist of one LTE resource block (one sub-carrier in one transmission interval) or one transmission interval for a frequency-selective cable in the HSPA standard. We observe one user located anywhere in the entire network area.
For each given user, one BS candidate serving was chosen from only one tier, according to certain criteria. Within the subsection B, which represents related work overview, we are studying some of the most commonly used criteria for selecting BS's candidate from all tiers and the criteria for selecting one serving BS from all the candidates serving BSs.

4) Serving tier, serving BS, and instantaneous SINR: As noted previously one candidate serving BSs is chosen by any of several criteria to be defined in B. out from all existing tiers as the serving BS. Thus, if there is only one accessible tier, the candidate serving BS from this tier is also the serving BS.

B. Related Work

The network topology, or spatial scheduling of the BSs, significantly affects the performance key parameters of the cellular networks such as the coverage and capacity. When we observe all through the main parameter, the SINR, which is a random variable in time from the receiver location perspective the layout of BSs represents the space of random events, i.e the distribution of the BS location belongs to the random process. Recently, it has become very popular to analyze the topological network in the form of Poisson Point Process (PPP), of which there are many interesting articles and very useful books [2]. Such modeling the BS cell site can describe irregular placements. Also, there are powerful tools of stochastic geometry [5] that can be utilized. The topology analysis in the PPP form commenced relatively long time ago [6] - [8]. However, the probability of the coverage of the signal and the average transmission rate based on Shannon's theory was derived recently [5]. Moreover, analysis of cellular networks with PPP distributed BSs has been extended to other network scenarios, including heterogeneous mobile networks [9] - [13], MIMO mobile networks [14], [15] and MIMO heterogeneous networks [14], [16], [17]. These papers do not consider systems with MIMO signal processing. The statistics of the propagation losses between a typical user and the BSs converge to the Poisson network model under i.i.d. shadowing with large variance [18]. However recent research demonstrate that these assumptions are quite restrictive and may not always hold in practice. Macro BSs are typically distributed more regularly than the realization of a PPP. However, this paper doesn't contain any research efforts devoted to investigating more accurate point process models for representing BS deployments. Such classes of point processes are the part of the future work [19].

C. Our Contributions

In previous work [3][4] authors made several analysis of trade-offs between deployment, spectral and energy efficiency based on multitier cellular networks in deterministic form (hexagonal).

In this work, we derive deployment vs. spectral efficiency (DE-SE) based on SINR metrics in cellular networks with PPP configured BSs for the first time. We used the relatively new approach to analysis of wireless networks trying to tackle the problem of SINR calculation at the receiver which is one and primary parameter for all interesting relations between power consumption spectral utilization and energy dispersal in wireless networks. This work belongs to unique research of trade-offs in wireless networks with a prime goal to derive one holistic utility function which will help operators to optimize the architecture and the topology of its networks. The main contributions of this paper are summarized in V.

II. System Model

We studied a network consisting of N_T, $k = \overline{1, N_T}$ tiers. Each tier is characterized as a homogeneous Poisson point process Φ_k with triplets $\{P_k, \lambda_k, \tau_k\}$, which denote the total transfer power in the k, the density of the access points AP and the SINR threshold below which the receiver cannot restore the content. Tiers are ranked in ascending order by the density of access points, i.e. $\lambda_1 \leq \lambda_2 \ldots \lambda_{k-1} \leq \lambda_k$. Given the density λ_k, the number of access points belonging to the layer k in the area A represents a random Poisson variable, with a mean density $A{\cdot}\lambda_k$, which is independent of other tiers. Furthermore, all access points in the k-tier have the same transmission power P_k.

Each downlink is affected constant path loss with exponent α and received signal from each AP is modeled by Rayleigh fading channel. With a sufficiently small thermal noise, this analysis can be reduced to considering interference from other RF signal sources only. Also, due to the simplicity of the analysis process, shadowing is neglected. For each tier, the factor of frequency reuse is one and the allocation of the RF band is according to the principle of orthogonality (which in principle means that one real channel is skipped between two tiers of the same standard). Therefore, at a typical user connected to tier k, the set of interfering APs include all the APs in tier k except the serving AP2. For the future work the results should be formed in such a way to enable spectrum sharing across tiers, lognormal shadowing, and any arbitrary fading distribution for the interfering signals.

We assumed following scenario [2]: the user is only allowed to access the BSs in tiers $1, \ldots, K_{open}$. For example, a macro-femto HCN with a mixture of open access (OA) and closed subscriber group (CSG) femtocells would be represented as an HCN with $N_T = 3$ and $N_{open} = 2$, with tier 1 representing the macrocells, tier 2 the OA femtocells, and tier 3 the CSG femtocells. Next, we assume that each BS in a tier transmits with the maximum power allowed for BSs in that tier. This immediately models the case of reference symbols (LTE) or pilot channels (HSPA), but also covers the case of data channels if we assume that the cells are all fully loaded.

We did not analyze the scenario which implies if the user moves quickly among service zones of two and more APs, which would cause that handoff occured [20].

III. Analytical Model

In order to demonstrate the relationship of deployment efficiency and spectral efficiency, it is necessary to firstly analyze the relationship between the network spectral efficiency and the total power in such a way that the base stations are distributed within the tiers in the form of PPP. The following analytical model consists of spectral efficiency for each tier, as well as of overall heterogeneous network's efficiency. Moreover, the model does not include thermal noise and considers path-loss exponent which is equal to 4.

Following the analytical models from [2], we observed two different cases: macro tier being the serving tier ($\mu_{c,i}i$) while pico tier being the candidate serving tier, and vice versa ($\mu_{c,j}$). The first case observes an arbitrarily located user served by micro tier, and the probability of its SINR to be greater than threshold can be expressed in the form presented by relation (1). In order to gain simplified model, η_{ABS} represents fraction of *almost blank subframes* (ABS), which are assumed to be subframes in which macro BSs do not transmit any power [2].

$$\begin{aligned}\mathcal{P}_i &= \mathcal{P}\left\{\Gamma_i > \gamma | R_i = r_i, R_j = r_j\right\} = \\ &= (1-\eta_{ABS}) \cdot \mathcal{P}\left\{\Gamma_i^{notABS} > \gamma | R_i = r_i, R_j = r_j\right\} = \\ &= 0.75 \cdot e^{-\pi\cdot\lambda_i\cdot r_i^2\cdot\sqrt{\gamma}\cdot\left[\cot^{-1}\left(\frac{1}{\sqrt{\gamma}}\right)+\beta\cdot\cot^{-1}\left(\frac{\pi\cdot\lambda_j\cdot r_j^2}{\beta\cdot\sqrt{\gamma}\cdot\pi\cdot\lambda_i\cdot r_i^2}\right)\right]}\end{aligned} \tag{1}$$

On the other hand, the probability that a user is served by pico tier, having the satisfactory level of SINR is presented within relation (2).

$$\begin{aligned}\mathcal{P}_j &= \mathcal{P}\left\{\Gamma_j > \gamma | R_i = r_i, R_j = r_j\right\} = \\ &= (1-\eta_{ABS}) \cdot \mathcal{P}\left\{\Gamma_j^{notABS} > \gamma | R_i = r_i, R_j = r_j\right\} \\ &+ \eta_{ABS} \cdot \mathcal{P}\left\{\Gamma_j^{ABS} > \gamma | R_i = r_i, R_j = r_j\right\} = \\ &= 0.75 \cdot e^{-\pi\cdot\lambda_i\cdot r_i^2\cdot\sqrt{\gamma}\cdot\left[\cot^{-1}\left(\frac{1}{\sqrt{\gamma}}\right)+\beta\cdot\cot^{-1}\left(\frac{\pi\cdot\lambda_j\cdot r_j^2}{\beta\cdot\sqrt{\gamma}\cdot\pi\cdot\lambda_i\cdot r_i^2}\right)\right]} \\ &+ 0.25 \cdot e^{-\pi\cdot\lambda_j\cdot r_j^2\cdot\sqrt{\gamma}\cdot\cot^{-1}\left(\frac{1}{\sqrt{\gamma}}\right)}\end{aligned} \tag{2}$$

A range expansion bias(θ) is applied to the mean received pilot powers at the user in order to favor offloading users to pico tier. In that manner, τ (relation (3)) is selection bias for tiers k. Also, β is a function of intensity ratios and transmit power ratios (4).

$$\tau = \sqrt[4]{0.063 \cdot \theta \cdot \frac{P_2^{tx}}{P_{1,reg}^{tx}}} \tag{3}$$

$$\beta = \frac{\lambda_2}{\lambda_1} \cdot \sqrt{0.063 \cdot \frac{P_2^{tx}}{P_{1,reg}^{tx}}} \tag{4}$$

The probability density functions of user's distance from serving tier is expressed as relation (5), while the same function for distance from candidate serving tier is presented by relation (6).

$$f_{R_i}(r_i) = 2.5 \cdot \pi \cdot \lambda_i \cdot r_i \cdot e^{-1.25\cdot\pi\cdot\lambda_i\cdot r_i^2} \tag{5}$$

$$f_{R_j}(r_j) = 2 \cdot \pi \cdot \lambda_j \cdot r_j \cdot e^{-\pi\cdot\lambda_j\cdot r_j^2} \tag{6}$$

Concise consideration of the previous statements converges to relations (7) and (8), which comprehend spectral deficiencies of macro and pico tier, respectively.

$$\mu_{c,i} = \frac{\lambda_i}{\ln 2}\int_0^\infty\int_0^\infty\int_{\tau\cdot r_i}^\infty \frac{f_{R_i}(r_i)\cdot f_{R_j}(r_j)\cdot\mathcal{P}_i}{1+\gamma} d\gamma dr_i dr_j \tag{7}$$

$$\mu_{c,j} = \frac{\lambda_j}{\ln 2}\int_0^\infty\int_0^\infty\int_{\frac{r_i}{\tau}}^\infty \frac{f_{R_j}(r_i)\cdot f_{R_i}(r_j)\cdot\mathcal{P}_j}{1+\gamma} d\gamma dr_j dr_i \tag{8}$$

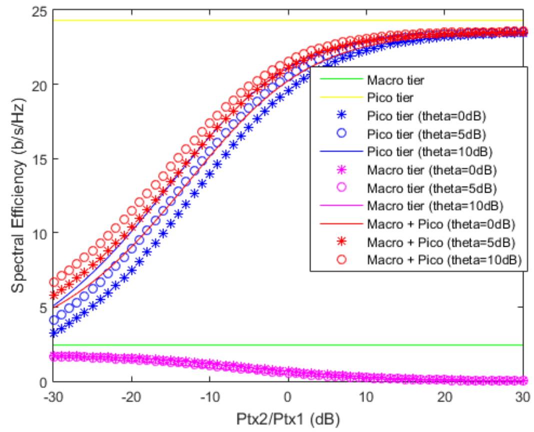

Fig. 1. Spectral efficiency vs. the relative transmit powers

Finally, the total spectral efficiency of heterogeneous network can be obtained by relation (9). Taking this analytical model into consideration, the Fig. 1 summarizes different scenarios which present spectral efficiency vs. relative transmit power.

$$\mu_{c,tot} = \mu_{c,i} + \mu_{c,j} \tag{9}$$

We can also draw similar conclusions to those obtained by authors from [2]. In this manner, from the spectral efficiency curves in Fig. 1, the benefits of spatial reuse of resources in a dense small-cell tier are significant. Comparing the scenarios of standalone pico and macro tiers emphasizes the strong potential of pico tier deployments, especially for the so-called green heterogeneous networks.

IV. Network Deployment Efficiency Analysis

Since analytical model was presented throughout the previous section, it is necessary to show the benefits and significant results that can be obtained using it.
Also, if we take the deployment efficiency into consideration, it has to be stated that this phenomena inevitably requires information about the total amount of cost of actual heterogeneous network deployment. Since our paper [4] provides relation between nominal power of considered BS and it's deployment cost, we present the model suitable for calculating the total deployment efficiency of heterogeneous network, in the form of relations (10) and (11).

Therefore, these relations are specified for the certain scenario of network deployment which envelopes two tiers ($N_T = 2$, pico and macro), each consisting of N_k BSs.
According to the relation (10), the total deployment cost for the engaged scenarios is presented in Fig. 2. The scenarios include the calculation of deployment cost for the certain value from a contemplated range of relative transmit power.

Accordingly, under the rule presented by relation (11) the total deployment efficiency is shown in Fig. 3, observing three different cases which differ in θ value. Finally, the

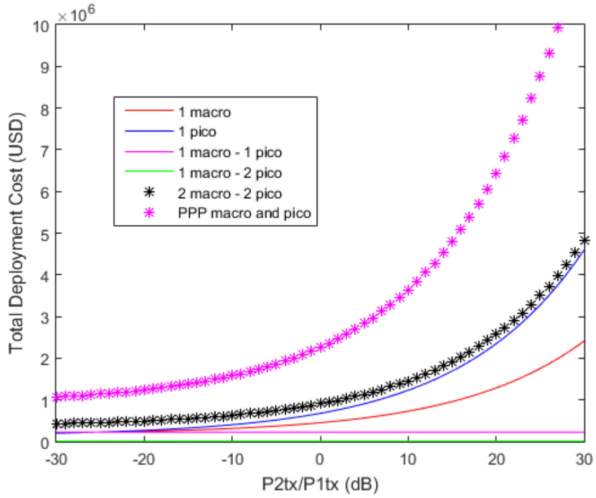

Fig. 2. Total deployment cost vs. the relative transmit powers

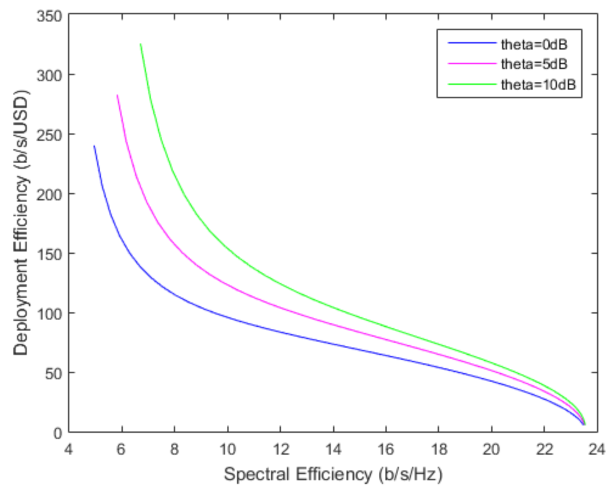

Fig. 4. Deployment efficiency vs. spectral efficiency

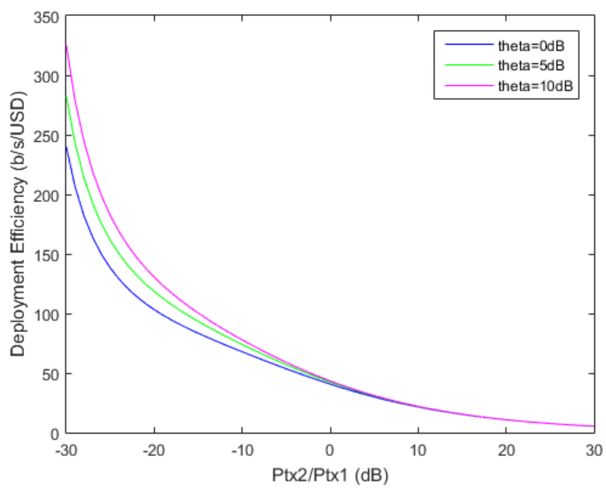

Fig. 3. Deployment efficiency vs. the relative transmit powers

trade-off between deployment and spectral efficiency is shown within Fig. 4.

$$C_{tot} = \sum_{k=1}^{N_T} \sum_{m=1}^{N_k} C_{BS_{k,m}} = \sum_{k=1}^{N_T} \sum_{m=1}^{N_k} \left(a \cdot P_{BS_{k,m}}^{b} + c \right)$$
$$N_T = 2, \quad N_k = \{N_1, N_2\} \tag{10}$$
$$N_1 = \lambda_i, \quad N_2 = \lambda_j$$
$$\{a, b, c\} = \left\{8.807 \cdot 10^4, 0.305, -4.53 \cdot 10^4\right\}$$

$$DE = \frac{B \cdot \mu_{c,tot}}{C_{tot}} \tag{11}$$

V. Simulation Results

Within this subsection, we will briefly present the background of the simulation set in ns3 simulator environment. The main idea was to inspect whether the simulation results can follow the presented analytical model or not. Therefore, three different scenarios were created in order to collect necessary data about SINR values at user terminals. Subsequently, being familiar with SINR results enabled the calculation of the spectral efficiency of heterogeneous network in a certain scenario.

Namely, the positions of BSs were determined by following the PPP, and their presentation is visible in Fig. 5. Also, the picturesque presentation of BSs' radiation, including one macro and ten pico BSs, is given within radio environment map (REM) shown in Fig. 6. Hence, all scenarios analyzed the distribution of SINR at user locations, as well as spectral efficiency, considering different transmit power levels for the pico tier, keeping the macro tier transmit power as a constant. It can be concluded that spectral efficiency has a growing trend in relation to relative transmit power. Also, the scenarios with greater number of pico cells within network deployment inevitably lead to higher spectral efficiency, which roughly follows the adopted analytical model.

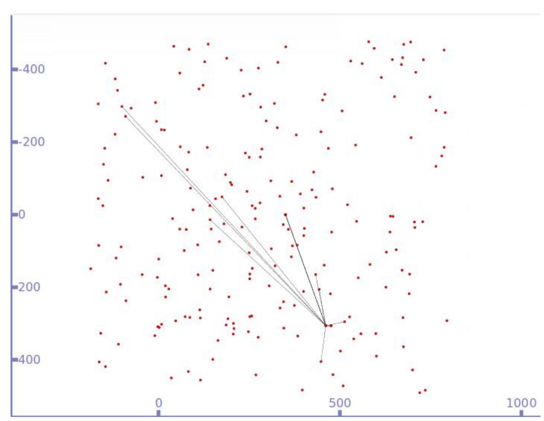

Fig. 5. Macro and pico tier BSs locations

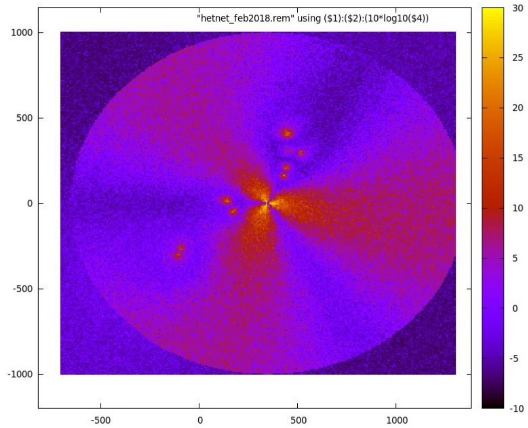

Fig. 6. Radio environment map (one macro and ten pico BSs)

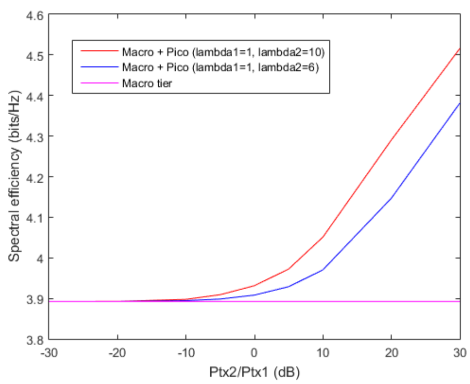

Fig. 7. Spectral efficiency vs. the relative transmit powers obtained from simulation

VI. Conclusions

The heterogeneous network concept in many respects meets customer requirements and maintains high energy efficiency. In this paper, we mainly investigate the impact of deployment efficiency of heterogeneous small cells BSs compared to the advantages of increasing the performance of wireless networks as a whole. Opposite to our previous work we used novel and very popular modeling the BS distribution over the heterogeneous multitier network as the Poisson Point Process. It is quite satisfying seeing that the results match both with the current state-of-art research articles and with our previous achievements. However it is early to say that by this work we achieved any major contribution since there is a lot of room for further investigations especially taking more complex scenarios under consideration and using enhanced models of PPP. The main achievement is that we made much easier analysis using such an approach and also simulation in NS3 is quite more tractable and comparable to an analysis. In summary, this analysis shows that the introduction of small cells in heterogeneous networks with any chosen distribution of base stations has a major impact on energy efficiency, and spectrum efficiency with acceptable cost reduction (it has the limitations as shown in the paper). The introduction of the small cells, i.e. pico cells, obviously increase the initial investments (Fig. 2), but analyzing the efficiency of network deployment in which the pico BSs are located, we obtained satisfactory results from which we conclude that deployment efficiency is in the upward trend until the total strength of the pixels is lower than the maximum power of one macro stations.

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