Outage Probability Determining for Wireless Systems in the Presence of Beaulieu-Xie Fading and Co-channel Interference Rayleigh Modeled

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***Abstract* - For realistic assessment of any wireless communication system performance an accurate modeling of this system is necessary. Till now, no many distributions were found to be able to characterize a channel with both, the line of sight (LOS) and the non-LOS (NLOS) components. One of such distribution unifying the non-central chi, generalized Rician and κ – μ distributions into a single fading distribution is Beaulieu-Xie distribution. Beaulieu-Xie fading, as disturbance in wireless system, will be considered in this work. Beside fading, co-channel interference (CCI) modeled by Rayleigh distribution, is present. An analysis of outage probability for selection combining (SC) receiver with multiple branches, used to mitigate these disturbances in the system, will be carried out. The expressions for probability density function (PDF), cumulative distribution function (CDF) and outage probability will be derived. A few graphs will be plotted and based on them an impact of fading and CCI parameters will be highlighted.**

Keywords - Beaulieu-Xie fading; outage probability; Rayleigh co-channel interference; selection combining

I. INTRODUCTION

The fading is the phenomenon of fluctuation in the amplitude of the sent signal [1]. Multipath is the appearance of non-line-of-sight (NLOS) signals at the receiver with a sufficiently short delay that it leads to an undesired distortion of useful signal. Furthermore, it is important to find as accurate a way as possible to describe this phenomenon in wireless systems.

Known distribution as are the Ricean and Nakagami-*m* have benefits as fading models, but also some lacks [2], [3]. For example, lacks are because of reflections and shadowing in the case of Ricean model and both, line-of-sight (LOS) and NLOS components in the event of Nakagami-*m* model. Because of that is necessary to develop new, powerful models in order to constrain the errors induced by fading.

To exceed the gap between theoretical considerations and measurements performed in a fading environment, intensive investigations of new general fading distributions are being carried out. That's why new fading model was introduced by Beaulieu and Xie [4] that overcomes these requirements and simultaneously includes both these models. BX fading model contains advantage of Nakagami-*m* model thanks to its flexible fading parameter and advantage of the Ricean model since its use of a non-central chi-distribution allowing description of both, LOS and NLOS fading channels.

Finally, this is general distribution because it includes some others known distribution as κ − μ distribution, generalized Rician, non-central chi, and a few others included in the said [4]. The BX distribution includes some special cases for different values of *m* and λ. Figure 1.3 in [5] shows the relationship between the BX distribution and other unimodal distributions [6].

It is still important to stand out that BX model can successfully describe the indoor propagation scenario in femtocells as well as high-speed train channels applied in 5GB networks. The statistics of the BX fading model, and later shadowed BX fading model, can be of significant application in land-mobile- satellite (LMS) and unmanned aerial vehicle (UAV) channels, which communicate via LOS signals. From that point many papers deal with this distribution for introduce fading influence.

So, in [7], the effect of randomness provoked by the Beaulieu-Xie fading on the performance of physical layer security is considered and that to the secrecy outage probability (SOP) and average secrecy capacity (ASC). Further, in [8], the second order characteristic of the BX fading channel were obtained, while in [9], [10] the asymptotic tight bound for correlated BX channel and different diversity combining techniques were determined.

The space diversity is very efficient method for reduction fading [1]. In [11], the first order system performance (outage probability (P_{out}), amount of fading (AF), average symbol error probability (ASEP) and channel capacity) for coherent and non-coherent modulation schemes) were derived for the Beaulieu-Xie fading channel with maximum ratio combining (MRC) diversity.

In wireless systems, the co-channel interference (CCI) can occur. CCI appears when a few devices are working on the same frequency channel. Consequently, it also should be introduced in modelling of wireless systems and after that in combating their influence. In paper [12], by determining the outage probability of several diversity models, they are compared analytically for an environment in the presence of Rayleigh fading and limitations from interference.

Even though selection diversity combining is inferior to optimum - maximal ratio combining, and equal gain combining, it is surely simpler for realization and implementation, and also cheaper. With SC combiner with more branches is used, quite good results are achieved in reducing the influence of fading and CCI.

In this work, we will consider Beaulieu-Xie fading, disturbing wireless system in the presence of CCI modeled by Rayleigh distribution. The outage probability for selection combining (SC) receiver with multiple branches will be determined, and the effect of diversity and the fading parameters on the performance measure will be demonstrated.

First section is introduction into new fading model with a review of the first works introducing BX fading. In the second section, system configuration is considered and main performance derived. In the third section obtained graphics are presented and parameters' analysis done. At the end is conclusion with promised future work.

II. MODEL OF CONSIDERED SYSTEM CONFIGURATION

We consider L-branch SC receiver in a channel disturbed by BX fading and Rayleigh CCI. The block-model of observed system is shown in Fig. 1. There are L input branches in SC receiver with input signals: x_1, x_2, …, x_n, $n \in \{2,\ldots, L\}$, and output signal is x. The CCI signals appearing at the receiver inputs are denoted by: $y_1, y_2, \ldots, y_n$, while the corresponding output signal is y.

Here, a SIR-based mathematical modeling of SC receiver will be made. The desired signal to CCI ratio (SIR) at each of L input branches in SC receiver is $z_i = x_i/y_i$. By working algorithm of multi-branch SC combiner, the output SIR z will be maximal value of all z_i: $z = \max(z_1, z_2, \ldots, z_i, \ldots, z_L)$.

Each desired signal at the input of SC receiver has the probability density function (PDF) modelled by the BX distribution [4]:

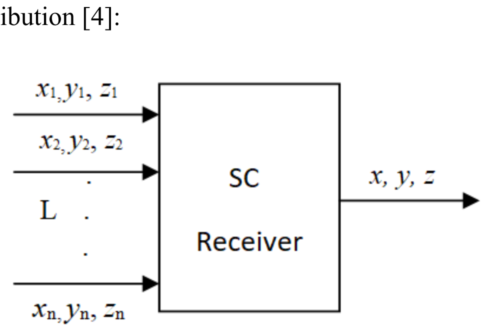

Figure 1. The block-diagram of the L-branch SC receiver

$$p_{X_i}(x_i) = \frac{2m x_i^{\,m}}{\Omega_i \lambda^{m-1}} \mathrm{e}^{-\frac{m}{\Omega}\left(x_i^{\,2}+\lambda^2\right)} I_{m-1}\left(\frac{2m\lambda}{\Omega_i} x\right). \tag{1}$$

To represent the PDF of the signal at each of the receivers' inputs as a sum, for the modified Bessel function of the first kind and order ν, $I_\nu(\cdot)$, can be used the shape [13; 17.7.1.1]:

$$I_\nu(x) = \sum_{k=0}^{\infty} \frac{(x/2)^{\nu+2k}}{k!\,\Gamma(\nu+k+1)}, \tag{2}$$

where Γ(γ) stands for Gamma function. This form leads to the signal's PDF:

$$p_{X_i}(x_i) = 2\mathrm{e}^{-\frac{m}{\Omega}\left(x_i^{\,2}+\lambda^2\right)} \sum_{i=0}^{\infty} \frac{\lambda^{2i} x_i^{\,2i+2m-1}}{i!\,\Gamma(i+m)} \left(\frac{m}{\Omega_i}\right)^{2i+m}, \tag{3}$$

where m present the fading severity parameter; λ is non-centrality parameter, and Ω_i is the power.

The CCI is modeled by Rayleigh distribution [14]:

$$p_{Y_i}(y_i) = \frac{2y_i}{s_i} e^{-\frac{y_i^2}{s_i}}, y_i \geq 0. \tag{4}$$

In (4), the CCI power is marked by s_i.

Let's derive the PDF of the SIR z_i from [15]:

$$p_{z_i}(z_i) = \int_0^\infty y_i p_{X_i}(z_i y_i) p_{y_i}(y_i)\, dy_i. \tag{5}$$

By replacement (3) and (4) in (5) and some mathematical manipulations, it is obtained:

$$p_{z_i}(z_i) = 2 \cdot \sum_{i=0}^{\infty} \frac{\lambda^{2i} z_i^{\,2i+2m-1} m^{2i+m} \mathrm{s}_i^{\,i+m} \Gamma(i+m+1)}{i!\,\Gamma(i+m)\,\Omega_i^{\,i-1} \left(\Omega + \mathrm{s}_i m z_i^{\,2}\right)^{i+m+1}} \mathrm{e}^{-\frac{m\lambda^2}{\Omega}}. \tag{6}$$

Now, it is necessary to calculate the cumulative distribution function (CDF) of the SIR z_i. Based on formula for CDF [14]:

$$F_{z_i}(z_i) = \int_0^{z_i} p_{z_i}(t)\, dt, \tag{7}$$

the CDF of z_i will be:

$$F_{z_i}(z_i) = 2\sum_{i=0}^{\infty} \frac{\lambda^{2i} m^{2i+m} \mathrm{s}_i^{\,i+m}}{i!\,\Omega_i^{\,i-1}} \cdot \frac{\Gamma(i+m+1)}{\Gamma(i+m)\,\mathrm{e}^{\frac{m\lambda^2}{\Omega}}} \int_0^{z_i} \frac{t^{2i+2m-1}}{\left(\Omega + \mathrm{s}_i m t^2\right)^{i+m+1}} dt. \tag{8}$$

The integral in previous expression can be solved by dint of the incomplete Beta function $B_w(a, b)$ [13] based on [16]:

$$F_{z_i}\left(z_i\right)=\sum_{i=0}^{\infty}\frac{\lambda^{2i}\Gamma\left(i+m+1\right)}{i!\Gamma\left(i+m\right)}\cdot\left(\frac{m}{\Omega_i}\right)^i \mathrm{e}^{-\frac{m\lambda^2}{\Omega}} B_{\frac{m\mathrm{s}_i {z_i}^2}{\Omega_i+m\mathrm{s}_i {z_i}^2}}\left(i+m,1\right). \tag{9}$$

Now, we should derive outage performance. Outage probability is defined as the moment at which the receiver power value goes under the threshold (where the power value relates to the minimal SIR). At that moment, the receiver is out of the range of base station in cellular communications.

The P_{out} can be simply calculated through PDF if we know PDF of SIR. The outage arises if the signal falls below the CCI power level. The derivation is based on integration over the PDF of SIR, and mathematically is equal to CDF from (7).

On the SC receiver output, P_{out} is [17]:

$$P_{out}\left(z\right)=F_{z_i}\left(z\right)=\left(F_{z_i}\left(z_i\right)\right)^L \cdot \tag{10}$$

For our case, after replacement of (9) into (10), we obtain P_{out} in the form:

$$P_{out}\left(z\right)=\left(e^{-\frac{m\lambda^2}{\Omega_i}}\sum_{i=0}^{\infty}\frac{\lambda^{2i}}{i!}\cdot\frac{\Gamma\left(i+m+1\right)}{\Gamma\left(i+m\right)}\left(\frac{m}{\Omega_i}\right)^i B_{\frac{m\mathrm{s}_i {z_i}^2}{\Omega_i+m\mathrm{s}_i {z_i}^2}}\left(i+m,1\right)\right)^L \cdot \tag{11}$$

III. ANALYSIS OF THE SYSTEM PERFORMANCE

To analyze the impact of fading and CCI parameters on the concerned measure, here P_{out}, we have plot some curves in the next Fig. 2. We assumed the minimal correlation between the input branches in the SC receiver. The powers are equal for all curves. The figure is made using Wolfram Mathematica and Origin.

We can conclude from the graph that due to the increase of the parameters m and λ, the outage probability decreases, and the system has better performance.

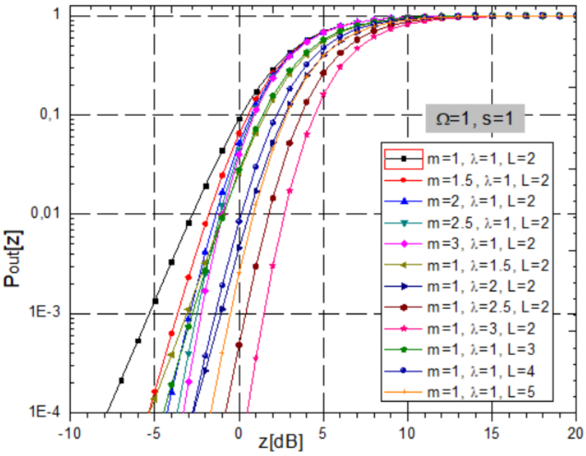

Figure 2. Normalized P_{out} of L-branch SC receiver versus output SIR, with variable values of fading parameters m and λ, and number of branches L

TABLE I. NUMBER OF TERMS THAT SHOULD BE ADDED IN EXPRESSION (11) FOR POUT TO REACH ACCURACY AT 5TH SIGNIFICANT DIGIT

	z=-10 dB	**z=0 dB**	**z=10 dB**
m=1, λ=1, L=2	5	5	8
m=1.5, λ=1, L =2	5	6	9
m=2, λ=1, L =2	5	7	11
m=2.5, λ=1, L =2	5	8	12
m=3, λ=1, L =2	5	10	14
m=1, λ=1.5, L =2	5	6	11
m =1, λ=2, L =2	5	8	16
m =1, λ=2.5, L =2	5	9	20
m =1, λ=3, L =2	5	6	24
m =1, λ=1, L =3	5	5	8
m =1, λ=1, L =4	5	5	8
m =1, λ=3, L =5	5	5	9

Also, with an increase of number of input branches in SC receiver, L, the P_{out} decreases what improves system performance. In massive MIMO systems, with a huge number of antennas at the base station, the usage of spatial diversity leads to phenomenon named "channel hardening" [18]. This means that a fading channel behaves as a non-fading channel. But, the fact is that biggest improvement is achieved with enlarging the number of branches from 2 to 3, and the improvement diminishes as L increases, and soon cannot be considered justified.

The number of terms that should be added in expression (11) for P_{out} in order to reach accuracy at 5th significant digit is presented in Table I. The parameters m and λ, as well as the number of branches L are changing, while powers take values: $\Omega=1$, $s=1$.

From this table one can notice that for small signals, i.e. for z=-10db, the series converges quickly, and for any value of m, λ, or L, it is enough to add only 5 additions in sum in (11). On the other side, for z=0 and z=10 dB, the number of elements in the sum increases and depends on the value of the fading parameters. When the parameter m and parameter λ increase, it is necessary to add a larger number of terms, because the series converges more slowly. The most additions need to be added is 24 for large signals, z=10db, when λ increases to 3.

IV. CONCLUSION

One of very important measure showing system reliability is outage probability. We determined P_{out} for severely degraded transmission conditions influenced by multipath BX fading and Rician CCI in mobile systems. Mitigation of fading influence was made by L-branch SC diversity receiver.

Since the mobile system performs better for the lower P_{out}, based on the analytical results and graphics of P_{out} versus SIR, we have noticed that wireless system works better for bigger number of branches. This means possibility of receiver to chooses the antenna with the biggest SIR to forward to the user, what matches the theory.

On this spot, one can notice that increasing of fading parameter m to infinity means a channel without fading, and leads to better system performance. Increasing of λ means increasing of useful signal power, what also improve system performance.

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