We are living in the digital world. Are universities actually ready to educate the future experts? Are we ready to teach our students for the future globalized competitive labor market for the jobs that today don’t exist? Zagreb School of Economics and Management started the new four year undergraduate program Business Mathematics and Economics for challenging this issue. The main point is in educating the experts who will fill the gap between business practice and the developing digital technologies and STEM. We are teaching business mathematics through case studies, real data and optimization algorithms. How do we do it? In this work we will present some case studies from our university practice where we use the technology for more dynamic mathematics class.

Keywords - digital technology; business mathematics; case studies; dynamic math class

I. INTRODUCTION

Zagreb School of Economics and Management (ZSEM) started a new undergraduate program Business Mathematics and Economics (BME) four years ago to challenge the digital and STEM issues in business practice. Since the companies have to adapt to these trends and execute the digital transformation the question is do they have the right experts to do it. Our students should fill the gap between technical and business people since the companies have the real problems in the communication between these two worlds. BME includes the combination of mathematics, statistics, economics, business administration, finance and computer science. The point is that we teach all these subjects trough case studies, discussions, problems solved using computers and software, projects, presentations and internships.

We also conducted a survey among the mathematicians from the Croatian business practice in order to collect insights about the specific business practice’s needs. The results suggest the intensive use of digital technologies and open source software for problem solving. Besides this, the students should get soft skills which will enable them to be good communicators when they have to present their quantitative results to their colleagues from other fields.

Based on all these facts we developed the curriculum which includes an intensive use of digital technology for solving mathematical and economics problems. In the following chapters we will describe how this looks like in our university practice. In chapter II we will describe the problem in a way how we are describing it to our students. In chapter III we will present the solution procedure and the results with economic interpretations. In the chapter IV we will comment some students’ impressions, and in the chapter V we will give the conclusions.

II. RELATED WORK

There are not too many related works on this topic. Mostly of the scholars are writing about using computers in teaching mathematics in primary and high schools, but not teaching business mathematics. The paper [1] This paper discusses the impact of the Teaching and Mastering Calculus Computer Courseware (TEMACCC) on the results of the students in the subject Calculus. But the paper doesn’t discuss the exact application of computers. In [2] the authors did an experimental study and compared the effect of Computer Assisted Instruction (CAI) and Conventional Method on the performance of undergraduate students in Calculus at Indian institutions, but also without exact examples of the applications. In [7] the examples from business mathematics are given but for the high schools and pupils not having the deeper knowledge of business mathematics.

III. PROBLEM DESCRIPTION

As an example how to illustrate our way of teaching mathematical methods applied in economics, business administration and finance we are considering a case from supply chain management. The students were asked to solve the following tasks.

(1) Based on the data given in Table I model the demand function as a linear function using the linear regression in Excel.
(2) For which prices the model (1) has economic sense?
(3) Determine the inverse demand function.
(4) Determine the total revenue function as a function of demand quantity.
(5) Graph the total revenue function in Wolfram Alpha.
(6) Determine the demand quantity for which the total revenue is maximum and calculate this maximum revenue.
(7) Determine the price for the quantity from (6).
(8) Based on Table II model the total costs function as a linear function of production quantity using Excel.
(9) Determine the fixed and the unit variable costs.
Determine the profit function as a function of production quantity using the total revenue function (4) and total costs function (8).

Graph the profit function using Wolfram Alpha.

For which production quantities the company is profitable?

Determine the maximum profit and the production quantity for which it is obtained.

Determine the average costs function.

Graph the average costs function and comment its rise and fall (increase and decrease).

Determine the coefficient of elasticity of the profit function at the production level 25 and then 60. Comment.

Approximate the profit function with linear function at the production level 20 and comment.

### IV. SOLUTION OF THE PROBLEM

To model the demand function we used Excel. First we introduced the scatter graph based on the demand data, Figure 1. Clicking on one of these points we introduced the trend line on the figure with corresponding equation.

\[ p(q) = -1.715q + 96.389 \]  

In order to answer the question (2), we have to solve the following system of inequalities:

\[ p \geq 0, q(p) \geq 0 \]  

We used Wolfram Alpha. The result is on Figure 2. Thus the solution is the interval

\[ 0 \leq p \leq 56.20 \]  

Inverse demand function can be obtained with Wolfram Alpha and is \( p = 56.20 - 0.58q \). The revenue function as a function of demand quantity is now

\[ R(q) = p \cdot q = (56.20 - 0.58q) \cdot q \]

The graph of the revenue function is on Figure 3. We can see that the revenue is a quadratic function whose graph is oriented down words. It means that its extreme point is maximum point.

To find the maximum revenue we can use Wolfram Alpha, but also Excel Solver. In Wolfram Alpha the syntax is “find maximum -0.58q^2+56.2q”. Using Excel Solver we have the interface as on the Figure 4 and 5. Thus, till the demand quantity 48.45 the revenue increases while for higher quantities the revenue decreases. Since the price is determined as a function of quantity \( p = 56.20 - 0.58q \) it means that with the higher quantities it is lower and it makes the revenue lower too.
This discussion is usually very clear to the students due to the graph. The beauty of using computers is that now we can simulate and analyze the revenue behavior depending on the input changes (sensitivity analysis).

Using Linear Regression (Add – in Data Analysis in Excel) we are obtaining the total costs function (Figure 6), \[ C(q) = 9.6164q + 136.07 \] The fixed cost is \( C(0) = 136.07 \) and unit variable costs 9.61. Using Wolfram Alpha, we are calculating the profit function (Figure 7).

\[
P(q) = R(q) - C(q) = -0.58q^2 + 46.5836q - 136.07 \quad (5)
\]

The graph of the profit function is in the Figure 8. From this graph we can observe the quantity interval of company profitability and the maximum profit.

Calculation gives us the maximum profit 799.29 at \( q = 40.158 \).

The average costs function is \[
AC(q) = \frac{C(q)}{q} = \frac{9.6164q + 136.07}{q} = 9.6164 + \frac{136.07}{q}
\]

(the graph is in the Figure 9). The syntax is “plot \(9.6164+136.07/q\) for \(q=0.01\) to 80”. 
The coefficient of elasticity is obtained using Wolfram Alpha (Figure 10). The syntax is \( q/(-0.58q^2+46.5836q-136.07)*\text{derivative}(-0.58q^2+46.5836q-136.07) \). Now it’s easy to plug the values \( q=25 \) and \( q=60 \).

In order to approximate the profit function with the linear one we can use Wolfram Alpha and the instruction “series”. The syntax is “series -0.58q^2+46.5836q-136.07 at q=20 of order 1”. The graph of a linear approximation is the tangent line. The linear approximation is used in economics very often since it’s easy to analyze. The economic interpretation of the slope is the marginal value. In our case, the slope represents the marginal profit at the production level 20.

V. IMPLEMENTATION AND STUDENTS’ REACTIONS

BME study program is organized as a very applied mathematics and computer science in economics, business administration and finance.

Every topic that we teach has to include all these issues. And students like this.

At our web site, https://www.zsem.hr/en/study-programs/undergraduate/business-mathematics-and-economics/program/, one can see that at the first year we teach the foundations of mathematics, Mathematical Analysis and Linear Algebra. These courses are the classical courses offered at every mathematics program. The students develop their analytical skills and acquire the fundamental knowledge.

After the first year all the courses are applied and are using the computers, digital technology and, mostly open source, software. The point is to apply. The point is to teach students how to use all the technologies available today. The point is to make them able to solve any problem using the knowledge that they have and can find in their environment very quickly. We are preparing them for the jobs that today don’t exist. Thus, we have to teach them to be independent, self confident, agile, flexible, analytical, but also to give them the knowledge.

The material that we prepare for the students is uploaded on Moodle platform which is used by our students very often. They download the files on their laptops or the computers in our computer lab. In Figure 12 the evaluation results on the scale from 1 – 5 related to Moodle usage are presented. The students that answered the question are the students of the second year.
When we asked the students from the third year about the applicability of the courses, the evaluations were about 4 (Figure 13). We have to emphasize here that the courses are mostly offered with the work in the computer lab and the material on Moodle. The students are solving small projects and can use the computers wherever they think computer can help.

Also, the midterm exams are organized including the theoretical and practical part. While the theoretical part is a closed book examination, in the practical part they download the data sets or some other related files from Moodle and solve the problems using computers (similar to chapter II).

They are also asked to search on the internet for some topics and then to type their critical opinion in a word file which is submitted for evaluation at the end of the midterm exam. Here is one example from the course Optimization.

**Example.** A tool box costs $55 to produce. We are considering charging a price of between $150 and $180 for this tool box. For a price of $150, $160, $170 and $180, the marketing department estimates the demand for the tool box in the three regions where the tool box will be sold (see the table). Use Excel.

<table>
<thead>
<tr>
<th>Price</th>
<th>Demand (in 000)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Region 1</td>
</tr>
<tr>
<td>150</td>
<td>38</td>
</tr>
<tr>
<td>160</td>
<td>34</td>
</tr>
<tr>
<td>170</td>
<td>26</td>
</tr>
<tr>
<td>180</td>
<td>22</td>
</tr>
</tbody>
</table>

(a) Estimate the linear functions for demands as functions of price in all three regions.
(b) Determine the total demand function.
(c) Determine the profit function as the function of price.
(d) Maximize the profit using Excel Solver with the condition that the price we are searching for has to be between $150 and $180. Write the optimal solution and the maximum profit.
(e) Write the estimated demands for all three regions.
(f) Maximize the profit using Excel Solver with the condition that the price we are searching for has to be between $160 and $180. Write the optimal solution and the maximum profit.

Students like this kind of work and examination. They don’t have to learn by heart, they are more asked to apply the knowledge from the books and the internet in the problems from the real business practice.

We have many partners from the business practice providing us the data and the problems for our students. Very often the problems are created based on the real problems from the bank industry, retail industry, production, transportation, telecommunication, etc.

The students are offered the programming languages R and Python, and the elective course Machine Learning. The big accent is given to all the digital technologies that can be helpful in business practice.

Our students are going on international exchange and come back home with the opinion that they are very competitive to the foreign students because they study this interdisciplinary program. The classes are in English which contributes to this competitiveness for sure.

Students are also participating in administration of our Facebook page, https://www.facebook.com/zsem.bme/. We often post the problems that we are solving during the class to popularize the application of mathematics and computer science in business practice.

**VI. CONCLUSIONS**

Four years ago ZSEM started a new four year undergraduate program BME. The program is an interdisciplinary program including mathematics and computer science applied in economics, business processes and finance. This curriculum is a result of the market needs, ZSEM market research and consultations with the Croatian employers.
Following trends of business school digital transformation, the usage of computers, digital technologies and software is very intensive. Besides working in the computer lab and using open source software, we insist on using the digital technologies for solving mathematical problems applied in economics, business processes and finance.

The classes and exams are organized in the sense of strongly digitally oriented processes. The students show their satisfaction in the evaluations executed in the framework of assessment of learning process.

In this work some examples of how to use computers in teaching business mathematics are described, solved and interpreted in the implementation framework.

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REFERENCES


