Prototyp of an optical - electrical measuring system for the determination of the gravitational constant for teaching purposes

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***Abstract*—Newton's gravitational constant plays an outstanding role in physics and is the most imprecisely determined natural constant from an experimental point of view. To determine this constant, one uses a set-up according to Cavendish, the so-called Cavendish experiment. The aim of the present work is to reconstruct this experiment and to implement a suitable measuring system with automated data acquisition and evaluation. This system is to be used in the physics lecture as a demonstrator and didactic support. Two tasks were addressed and solved:**

1.) mechanical construction of a torsion balance
2.) automated measurement data acquisition and evaluation

This paper deals with the second part. The result of this work is a complete characterization of the mechanical properties of the torsion balance and the construction of a functional, automated optical-electrical measuring and evaluation system for determining the gravitational constant γ.

***Keywords*—Cavendish experiment, torsion balance, gravitational constant, optical-electrical measuring system, teaching experiment**

I. INTRODUCTION

Gravity is one of the most important natural phenomena in our universe and is one of the four basic forces in physics. It was first discovered by Sir Isaac Newton (1643 - 1727) in 1687 with a mathematical formula. It is known as Newton's law of gravitation and, together with the axioms of motion, forms the basis of Newtonian mechanics. Newton's law of gravitation describes in mathematical form the force relationship between two masses. Newton found out how the masses of bodies and their distance must be put in relation to each other so that the mutual force effect is correctly represented. The law contains a constant of proportionality which is neither dependent on the masses of the bodies nor on their distance. This constant is called the gravitational constant γ, and its determination is done experimentally. The value for γ is very small because the force between everyday things is present but not perceptible. Henry Cavendish first succeeded in determining the magnitude of this constant. This was realised in the so-called Cavendish experiment with the help of a torsion balance. The currently recognised value for the gravitational constant γ is [1]

$$\gamma = 6.67430(15) \cdot 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{s}^2}.$$

Within the scope of a project a torsion balance was constructed and realised at the Carinthian University of Applied Sciences according to the idea of the Cavendish experiment. Usually, the measurement with such an apparatus (recording and evaluation of the measurement data) is done manually, which means that a person who logs the data must be present during the entire measurement run. In this paper, an automated set-up for the entire experimental procedure is presented.

II. PROBLEM AND TASK DEFINITION

The measurement of the gravitational constant requires a very careful experimental procedure, as the effect of gravity is extremely weak. Already the smallest disturbances make the measurement unsuitable. The risk of the experiment being disturbed by the presence of people or even by the experimenter is significantly greater than a measurement in an isolated room. The first tests with the torsion balance proofed that the results are strongly distorted due to external influences. Vibrations or air draughts caused by moving persons in the room have a negative effect on the experiment. In high-precision measurements of the gravitational constant, these influences are completely excluded by the experimental setup [2], which is not possible in this work. But these impacts should at least be kept as small as possible. Therefore, the design of an automated measurement system is favourable. Due to the setup, a simultaneous data evaluation can also be carried out. This concept should make it possible to leave the room after the start of the experiment, which results in a reduction of disturbing influences, since no one has to intervene in the measurement operation any more. Furthermore, it is expected that the results will be more reproducible and have a higher accuracy due to the automated data aquisition and evaluation.

III. BASIC SETUP OF CAVENDISH EXPERIMENT

For a better understanding, this section briefly summarises the principle procedure of the Cavendish torsion balance [3]. It consists of two pairs of balls, where the balls of a respective pair attract each other due to gravity. The effective gravitational force is measured with the help of a so-called torsion balance. The basic layout of the system is shown in figure 1.

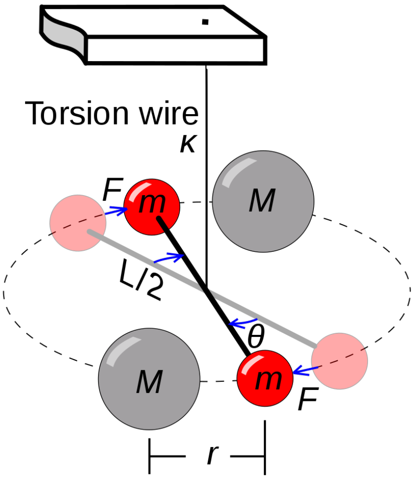

Fig. 1: Schematic diagram of a torsion balance for determining the gravitational constant [4]

At the beginning, the torsion balance is in an equilibrium position, this means that the torsional and the gravitational force mutually cancel out. By deflecting the heavy spherical masses M to a new position, the equilibrium between both force is disrupted and the movable small ball masses m, which are attached to the torsion wire, start to oscillate in a damped manner. After a certain time, the small masses m remain in a new equilibrium position. In this position, the effective gravitational force between the spherical masses and the torque of the torsion wire cancel each other out again. This approach and the period of the oscillations are used for the calculation of the gravitational constant. The task now is to detect the oscillations and the position of the small masses m as accurately as possible. A proper measuring module is needed for this purpose.

IV. SELECTION OF USED COMPONENTS

To develop a suitable measurement system, different measurement principles were first considered and compared in order to decide which measurement method would be appropriate for the job. In addition to suitability, the financial budget also played a role in the selection of components. Furthermore there was a self-imposed requirement to recycle as many existing components from broken devices as possible. Therefore a contact image sensor from a scrapped flatbed scanner was used as the sensor for the measurement system. This device has the ability to detect a laser beam with positional accuracy. MATLAB software was provided to evaluate the measurement data, as it is very useful to process and display large amounts of data with this programme. The STM32f407 microcontroller from STMicroelectronics was applied to control the sensor, process the measurement data and supply power to the measurement circuit. The interface between the microcontroller and the sensor is managed by an SD card module. With this concept, the measurement data can be stored without the aid of a computer.

Figure 2 shows the main components of the measuring system. In particular, one can recognise the sensor removed from the flatbed scanner. It is perfectly suited for the locally resolved detection of the laser beam. In a first step, the data sheet of the sensor had to be found. This is necessary to determine the timings and the pin assignment. The knowledge is also required to control and read out the device with the microcontroller. In addition, the electrical circuit can be taken from the data sheet, which is important to supply the pins of the sensor with the correct voltages and currents. To be on the safe side, the sensor signals were checked using an oscilloscope.

Figure 3 illustrates an example of the output signal after a proper electrical setup. A recorded period of the sensor output data is shown. The first part of the signal (yellow circle in the figure) indicates that the following segment contains the sensor information. In order to better visualize the functionality of the sensor, a box was placed on the sensor surface and covered it partially (red circle in the figure). The position of the object on the sensor surface is detected, which can be clearly seen in the output signal by the reduced amplitude at the location of the placement.

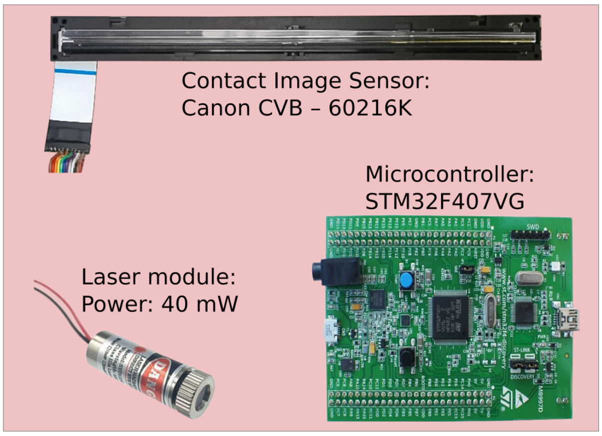

Fig. 2: Main components of the measuring system used

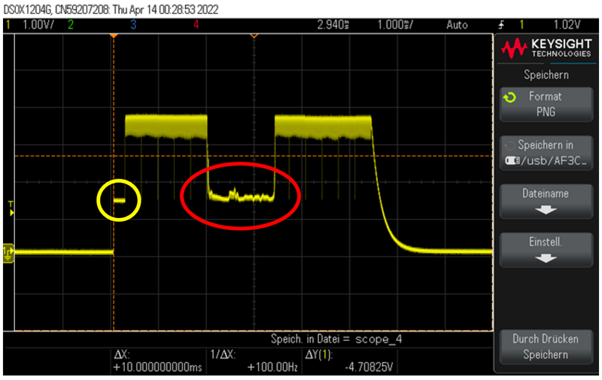

Fig. 3: Measured output signal of the partially covered image sensor demonstrating the functionality of the sensor

V. CONCEPT IMPLEMENTATION

The measurement signal of the Cavendish experiment (deflections of the masses) is very small and must therefore be amplified considerably. The amplification is best achieved by reflecting a laser beam from a mirror attached to the torsion wire and projecting it onto a distant wall. The displacement of the laser beam can be detected with different methods, namely manually, with the help of a camera or directly with a contact image sensor. The latter method provides the best and most accurate measurement results and is also significantly cheaper than a high-resolution camera system. The measurement signal recorded by the sensor is processed further with the microcontroller and evaluated by using a MATLAB script.

VI. RESULTS

In this work, the implementation of the measurement system and the determination of the inertia characteristics and the wire properties of the entire torsion balance have been realised. The final assembly and the main components of the measuring system are shown in figure 4. A detailed description of the construction is documented in bachelor thesis [5], which is published as part of the investigations. The central element is the torsion balance. As one can see, the laser is pointed to the mirror that is attached to the torsion wire and reflected onto the wall of the laboratory or the image sensor, depending on the experimental setup. The displacement of the reflected laser beam is tracked and logged either manually (note the measuring tape on the wall), by a camera system or by the image sensor. As already mentioned, the highest precision is achieved with the image sensor.

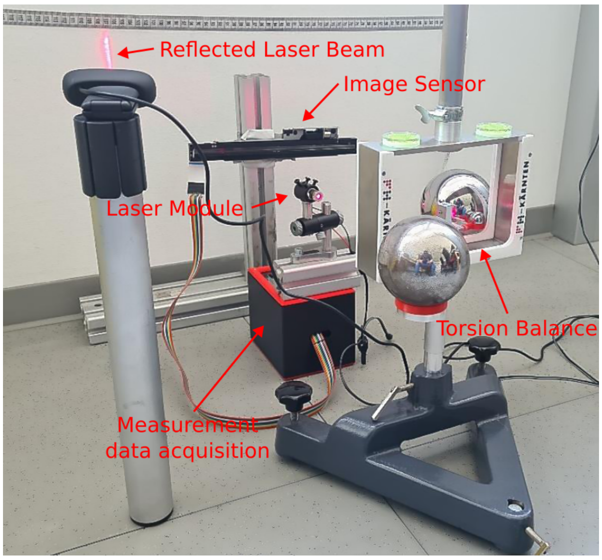

Fig. 4: Torsion balance and automated measurement system

Now the characterization of the torsion balance is carried out. This step is absolutely necessary to ensure an independent determination of the gravitational constant. Figure 5 shows an example of the measured oscillations, which were recorded with the automated measuring system.

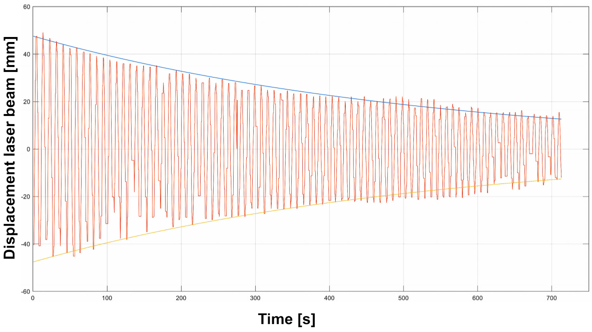

Fig. 5: Measured oscillations - tungsten wire thickness 0.2 mm

The evaluation of these oscillations for different test setups provide the parameters listed in TABLE I and TABLE II respectively.

One can clearly see the exponential decrease of the amplitude, which is typical for a damped oscillation. From the temporal intervals of the maxima, the average period durations were determined. With the help of these parameters, the geometry of the torsion balance and the masses of the moving spheres, the properties of the wire can be calculated as follows (e.g. [6] or [7]):

The moment of inertia of a sphere around its center of mass is given by the formula

$$J_{\text{sphere}} = \frac{2}{5} m_{\text{sphere}} r_{\text{sphere}}^2 \tag{1}$$

In this setup the mass of one sphere is $m_{\text{sphere}} = 32$ g, which is a typical size for mobile torsion balances[1]. The balls are positioned on the torsion balance at a distance of d offset from the wire (axis of rotation). Therefore, according to the parallel axis theorem the modified moment of inertia of the sphere can be calculated as

$$J'_{\text{sphere}} = J_{\text{sphere}} + m_{\text{sphere}} d^2 \tag{2}$$

Since the system consists of two spheres, this expression must be multiplied by a factor 2. In this way one obtains J_{w} and J_{s}. The following differential equation holds for a torsional oscillation:

$$J\ddot{\varphi} = -D\varphi \tag{3}$$

From this equation follows the relationship between moment of inertia, period of oscillations and directing moment. It results in

$$J_{\text{xx}} = \frac{1}{4\pi^2} T_{\text{xx}}^2 D, \tag{4}$$

where "xx" has to be chosen according to the experimental setup. With this approach the unknowns D and J_0 can be determined experimentally from the measured quantities T_{ow} and T_{os}. Note that the following relationship holds

$$\begin{aligned} J_{\text{ow}} &= J_0 + J_{\text{w}} \qquad \text{and} && (5) \\ J_{\text{os}} &= J_0 + J_{\text{s}} && (6) \end{aligned}$$

Abbr.	Meaning
J_{w}	Moment of inertia tungsten balls (shifted by 5 cm) without arm
J_{s}	Moment of inertia steel balls (shifted by 5 cm) without arm
J_{ow}	Moment of inertia arm + tungsten balls
J_{os}	Moment of inertia arm + steel balls
T_{ow}	Period duration arm + tungsten balls
T_{os}	Period duration arm + steel balls
J_{o}	Moment of inertia arm without balls
T_{o}	Calculated period arm without balls
D	Directing moment (material constant)
G	Shear modulus

TABLE I: Abbreviations of evaluated parameters

Now the constants J_{o} and D are known. These parameters were used to make a prediction for the period T_{o} in the case that the arm oscillates without balls. This period T_{o} was then used for comparison with the actual measured period $T_{\text{o, measured}}$.

A summary of the calculated results can be seen in the following TABLE II

Experimental Characterisation of the System
$J_{\text{o}} = 7.275 \cdot 10^{-6}$ kg m^2
$J_{\text{ow}} = 1.689 \cdot 10^{-4}$ kg m^2
$D = 8.779 \cdot 10^{-5}$ kg/m^2
$J_{\text{os}} = 5.005 \cdot 10^{-5}$ kg m^2
$T_o = 1.809$ s
$G = 167.684$ GPa
$T_{\text{o,measured}} = 1.885$ s
$G_{\text{literature}} = 158$ GPa
$\Delta T_o = 4.22\%$
$\Delta G = 6.13\%$

TABLE II: Results of wire and system properties

The percentage difference between the measured period $T_{\text{o,measured}}$ and predicted period T_{o} is 4.22% (1.885 s to 1.809 s). The shear modulus G was also determined [8] using the first setup and compared with the literature value. The deviation of the literature value of the shear modulus G from the calculated value is 6.12% (158 GPa to 167.684 GPa). The small differences between the results can be attributed to uncertainties in the wire measurement (thickness and length), for example.

VII. SUMMARY AND OUTLOOK

In this contribution, a torsion balance was successfully equipped with an optical-electrical measurement and evaluation system. Essential for the determination of γ is the sole effect of the gravitational force. Unfortunately, the available spheres have strong magnetic properties (proven with a magnetic field meter) and are therefore unusable for the actual experiment. In the next step, new spheres (made of a non-magnetisable material, e.g. lead) must therefore be installed. Thus, nothing stands in the way of determining γ and using the experiment in teaching.

VIII. ACKNOWLEDGEMENT

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REFERENCES

[1] Codata value: Newtonian constant of gravitation. [Online]. Available: https://physics.nist.gov/cgi-bin/cuu/Value?bg

[2] J. H. Gundlach and S. M. Merkowitz, “Measurement of Newton’s constant using a torsion balance with angular acceleration feedback,” *Phys. Rev. Lett.*, vol. 85, no. 14, pp. 2869–2872, 2000.

[3] M. SHAMOS, *Great Experiments in Physics. Edited by M.H. Shamos, Etc.* New York, 1960. [Online]. Available: https://books.google.at/books?id=uU4OMwEACAAJ

[4] Wikipedia contributors. Cavendish experiment. [Online]. Available: https://en.wikipedia.org/wiki/Cavendish_experiment

[5] P. Lippitz, *Konstruktion einer teilbaren Gravitationswaage mit Messsytem.* Carinthia, Austria: Carinthia University of Applied Sciences, 2022.

[6] D. C. Giancoli, *Physics: Principles with applications*, 7th ed. Upper Saddle River, NJ: Pearson, 2013.

[7] H. J. Paus, *Physik in Experimenten und Beispielen*, 3rd ed. Munich, Germany: Hanser, 2007.

[8] H. J. Eichler, H.-D. Kronfeldt, and J. Sahm, *Das neue Physikalische Grundpraktikum*, 3rd ed. Wiesbaden, Germany: Springer, 2016.

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[1] For the mass of the heavy balls, 1.5 kg has turned out to be a good choice.