# Implementation of a Facilitative Approach to Teaching Mathematical Disciplines to Future Mathematics Teachers by Means of the MAPLE Package

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***Abstract* - The aim of the article is to reveal the experience of using a facilitative approach in teaching future teachers of mathematics in teaching mathematical disciplines using the mathematical package MAPLE. Among the methods of studying the effectiveness of the facilitative approach in teaching mathematical disciplines to future teachers of mathematics, which highlights the advantages and disadvantages of this process, we have chosen SWOT-analysis. We have determined experimentally that one of the appropriate technologies of the facilitative approach when studying mathematical analysis by future mathematics teachers is "World Cafe". The description of its practical possibilities confirms its effectiveness in creating favorable conditions for training future mathematics teachers in mathematics analysis classes using the MAPLE package, to increase the motivational component of educational activities, creative personality and subjective involvement in working on problems and situations.**

***Keywords - facilitative approach; teaching future teachers of mathematics; teaching mathematical disciplines; mathematical package MAPLE.***

## I. Introduction

The current socio-cultural and educational situation in Ukraine is characterized dynamism, uncertainty, variability and low predictability. Under such circumstances the role and importance of professional training of future professionals in various social areas. Educational reform in Ukraine is aimed at humanizing education, democratic and tolerant attitude towards children, ensuring their right to choose the educational trajectory, meeting individual needs and desires, developing the capabilities and abilities of each child [4; 20]. In this context, the development of the New Ukrainian School uses the experience of the functioning of educational systems of the most progressive countries in the world, learning in which is based on the pedagogy of partnership [11; 15]. In this regard, the issue of appropriate training of teachers in the higher education system is relevant.

These fundamentally new conditions put forward an improvement in the quality of education professional training of future teachers of mathematics, able to operate in today's conditions. There is an application of a facilitative approach to the professional training of future mathematics teachers relevant given the need to use interactive learning technologies.

### *A. How the educational activities of future teachers should be organized*

The organization of educational activities of future teachers should combine two important components: on the one hand – subject-oriented approach (learner-centered approach), based on individual characteristics of each participant in this process, on the other – the formation of their ability to interact in a team to achieve a certain result and communicate in a multicultural environment. This leads to the search for and implementation of various interactive methods based on the technology of cooperation and partnership, which help to build joint learning activities, promote socialization of the individual, allow you to successfully master the relevant competencies. One of such methods, in our opinion, is facilitation, which changes the roles of teacher and student, makes them like-minded and equal participants in the pedagogical process.

In the system of professional education of future teachers there is a gradual accumulation of experience in implementing a facilitative approach to the educational process, however, in our opinion, in modern research of domestic scientists this issue is not given enough attention. In particular, it requires the study and understanding of the use of facilitative technologies in the teaching of mathematical disciplines and the development of their methodological content.

### *B. Formulation of research tasks and methods*

The purpose of the article: to reveal the experience of using the facilitative approach in teaching future teachers of mathematics in teaching mathematical disciplines using the mathematical package MAPLE.

A set of interrelated methods of pedagogical research was used to solve the formulated tasks in the research:

- theoretical: analysis, systematization and generalization of pedagogical and psychological research to clarify the content of the concept of "facilitative approach"; modeling to develop a model of mathematical analysis classes using the facilitative approach in teaching future teachers of mathematics in teaching mathematical disciplines using the mathematical package MAPLE;

- empirical: pedagogical observation of the educational process, analysis of the quality and success of training.

The methodological basis of the study are developed in world humanistic philosophy, psychology and pedagogy ideas about the self-worth of the individual and the inherent needs of continuous improvement, the position of unity of consciousness and activity, the idea of the relationship between pedagogical facilitation and universal values, meaning of life, love, freedom, creativity, dignity, the idea of the integrity of the individual. The provisions of pedagogy of self-development as a methodological basis of education and training, the provisions of system-synergetic, competence, personality-activity and eco-facilitative approaches to the organization of the pedagogical process were also system-forming for us.

### *C. Analysis of scientific sources*

Our study is relevant to the article by Jacobs J., Seago N., & Koellner K. [5], which describes a study of video facilitation resources for math teachers (PowerPoint slides, video clips and transcripts, lesson schedules, math assignments and other handouts, a field guide to geometric transformations, congruences and similarities, interactive computer applets, built-in scores and a comprehensive facilitator's guide). Research shows that this presentation of information in the teaching of mathematical similarity on the basis of geometric transformations has a positive effect on the knowledge of teachers of mathematics. Our research concerns the teaching of mathematical analysis, but we have obtained similar results.

Given the work [1; 7; 8], the role and importance of pedagogical facilitation in the training of future teachers of mathematics is that this type of professional and pedagogical activities of teachers provides pedagogical support for personal growth and pedagogical support of his educational achievements, actualizes students' desire for self-identification, self-development self-realization, self-improvement, promotes the implementation of individual characteristics in the study of mathematical disciplines (cognitive abilities, creativity, inner intentions, inclinations, abilities, talents, aspirations).

Also, our study can be used to build pedagogical technology to prepare future teachers for pedagogical facilitation. Our article complements the study of special training of future teachers for pedagogical facilitation, which is defined as a type of pedagogical interaction that provides conscious, intensive and productive development and self-development of its participants [13; 14]. The result of professional training of future teachers for pedagogical facilitation is appropriate training, which is defined as a stable integrative personal education, including professional motives, goals, general and professional knowledge and skills, personal qualities of the future teacher, providing productive interaction with students and providing them with effective pedagogical support.

Our research is also consistent with research on the application of mathematical packages in the training of specialists in mathematics, computer science and physics [2; 3; 6; 12; 17]. It is noted that modern mathematical education of future mathematics teachers is associated with the mastery of theoretical principles, methods and tools of computer science as the basis of teaching mathematics, increases the level of professional activity of mathematics teachers.

## II. THE EXPERIENCE OF USING THE MAPLE PACKAGE IN THE IMPLEMENTATION OF A FACILITATIVE APPROACH TO THE TEACHING OF MATHEMATICAL DISCIPLINES FOR FUTURE MATHEMATICS TEACHERS

### *A. Characteristics of pedagogical facilitation*

Based on the concept of K. Rogers [15], pedagogical facilitation we mean a special style of subject-subject interaction, due in particular to the personal qualities of the teacher-facilitator, which promotes personal activity of all subjects of the pedagogical process and ensures learning productivity. The teacher-facilitator plays the role of the unspoken leader of the group, who by his presence and influence helps each of its members to show initiative, makes the decision-making process easier and more efficient, provides positive interpersonal communication.

The defining characteristics of pedagogical facilitation are cooperation; own position; individuality and equality; self-disclosure; organization of educational space [18].

Under the facilitative approach in teaching is understood a set of principles of the facilitator and ways to implement them in educational interaction through certain algorithms and techniques. Among the properties of the facilitative approach, related to the content-target and structural features of facilitative interaction, are their subjective orientation, non-directive interactivity, contextuality, dynamic multitasking [16; 20].

The problem of applying facilitative approaches in the teaching of future teachers determines the search for innovative approaches in higher education, based on the latest learning technologies. The general scheme of implementation of the facilitative approach in teaching is presented in Figure 1.

The analysis of this scheme shows that the implementation of the facilitative approach has an algorithm similar to the algorithm for solving mathematical problems. This determines the feasibility of its use in the teaching of future mathematics teachers.

### *B. SWOT-analysis of the use of the facilitative approach in teaching*

Among the methods of studying the effectiveness of the facilitative approach in teaching mathematical

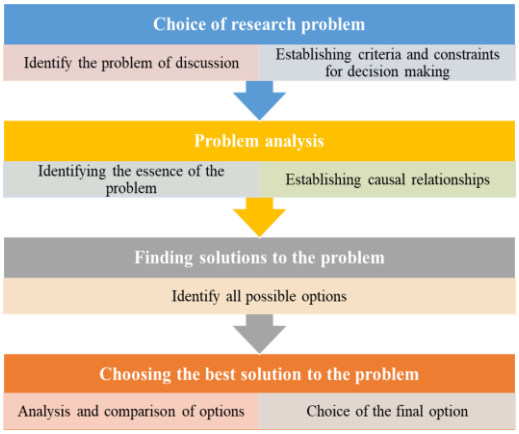


Figure 1. Stages of facilitation in learning

disciplines to future teachers of mathematics, which highlights the advantages and disadvantages of this process, we have chosen SWOT-analysis.

The results of our SWOT analysis are presented in Table 1.

This analysis determines the feasibility of using a facilitative approach in the teaching of future mathematics teachers.

Among the main characteristics of the implementation of the facilitative approach in the training of future teachers of mathematics, we highlight the following:

- directing the interaction of teacher and student to solve educational and creative tasks on the basis of cooperation and mutual understanding;

- taking into account the right of each participant to his own position and tolerant attitude to opinions other than his own;

- involvement of each student and teacher in the organization of joint activities;

- adoption of rules on ways of interaction in the group with simultaneous individual and group responsibility for the results of activities [15];

- directing the teacher's activities of students to make logically sound decisions, taking into account the specifics of the mathematical discipline in which the problem is considered.

TABLE I. SWOT-ANALYSIS OF THE USE OF THE FACILITATIVE APPROACH IN TEACHING

| BENEFITS | DISADVANTAGES |
|---|---|
| following the principle of cooperation<br>development of communicative culture of students<br>actualization of motivational resources for learning<br>independent search and processing of information on a particular mathematical problem<br>creating a favorable psychological climate for solving the problem, showing initiative and independence<br>creating a positive psychological climate of trust and support between teacher and students<br>use of interdisciplinary links<br>selection of information resources appropriate for solving the problem | increase the time to find a solution to the problem<br>the opinions of individual students may not be taken into account<br>the degree of involvement of students in solving the problem can vary significantly<br>the wrong role position of the teacher-facilitator is possible<br>specificity of mathematical knowledge |
| **OPPORTUNITIES** | **THREATS** |
| formation of skills of group activity, cooperation in decision-making<br>formation of facilitative competence<br>manifestation of their own intellectual abilities<br>development of creative thinking<br>development of attractive and assertive personality traits<br>improving the skills of solving competence tasks<br>improving digital skills | the emergence of discomfort in communication between the participants in the process<br>possible loss of the solution option or no solution at all<br>the problem-solving process can turn into a facilitator's own presentation |

### *C. Survey of teachers of fundamental disciplines in the specialty "Mathematics" of pedagogical universities*

We conducted a survey on the use of a facilitative approach in the training of mathematics teachers. 28 mathematics teachers took part in the survey. According to the survey results, only 18.7% of teachers use facilitative approaches in all types of classes, 28.6% indicate that they do not take into account the provisions of this approach in their professional activities, and 52.7% of teachers indicate that they partially use it only for time of practical and seminar classes. 54.8% of teachers turn to additional literature (articles and manuals) for the implementation of the facilitative approach, another 37.3% do it sporadically use Internet resources (video) for this purpose. Proportionality in the answers about teachers' preferences for the introduction of a facilitative approach in different types of classes was interesting: 34.6% of respondents prefer the introduction of a facilitative approach to study theoretical issues, and during the training of practical skills 65.4% turn to a facilitative approach. The results confirmed the fact that it is currently difficult to find a teacher who does not use IT in their practice. Yes, to the question: "Do you use IT in your work?" none of the respondents gave a negative answer. 76.9% of participating teachers answered that they use IT quite often, 23.1% - use, but not systematically. These results indicate that teachers in Ukraine today are well aware of existing information technologies and educational resources. All surveyed teachers used in their practical activities presentations and demonstration materials (interactive posters, diagrams, drawings), more than half use videos, interactive practical work, multimedia systems. Almost the same number of respondents also deal with methodological collections and electronic textbooks. One third of the teachers answered that they used the Maple mathematical package in practice.

Thus, the analysis of the results of the survey conducted among teachers revealed that respondents use a facilitative approach in their practical activities and consider its use necessary today. More than two thirds of respondents believe that this process is effective and ready for its implementation.

### *D. Describe the experience of using a facilitative approach in teaching future teachers of mathematics when teaching a course of mathematical analysis by means of Maple*

The facilitative approach to the training of future mathematics teachers can be carried out in a group or microgroups in the process of verbal exchange of each member of the group or individual microgroups whose members perform a common task. At the same time, the application of this approach is expedient both during lectures and in seminars and practical classes in mathematical disciplines, and the direct choice of the appropriate facilitative technology determines the following aspects: the purpose and objectives of joint activities to achieve the educational goal; algorithm of performance of work and forms of cooperation of participants at its various stages; ways to organize feedback; list of involved resources and digital tools.

Another factor in the successful implementation of a facilitative approach in the teaching of mathematical disciplines to future teachers of mathematics, we consider the use of computing power of information technology.

Analysis of the work environment and the realities of modern mathematical training show the feasibility of studying the possibilities of using several packages, but the limited study time and financial costs narrow the range of SCM that should be considered in higher education. The power of symbolic calculations and graphics, a wide range of teams that support the solution of problems in various fields of mathematics led to the choice of mathematical environment MAPLE [9; 19].

We describe the experience of using a facilitative approach in teaching future teachers of mathematics when teaching a course of mathematical analysis.

The course "Mathematical Analysis" is studied by students in the first and second years of pedagogical universities, and in the third and fourth years they use the acquired knowledge to study courses of complex analysis, differential equations, equations of mathematical physics, special sections of mathematical analysis. The material of the course of mathematical analysis has a powerful potential for the systematic formation of their readiness for teaching.

The organization of the educational process at the Pedagogical University for students to study the course "Mathematical Analysis" involves the implementation of two main components: the study of theoretical material and its application to solve problems. When studying the course "Mathematical Analysis" at the Pedagogical University, the student must master a certain system of knowledge and skills on the main issues of the course (master the system of concepts and master the apparatus of differential and integral calculus, learn to use knowledge to solve abstract and applied problems). In addition, students should develop mathematical abilities, form a high level of mathematical thinking. Therefore, there is a need to improve the teaching methods of mathematical analysis, namely to apply the technologies of the facilitative approach.

Note that the correct supervision of the facilitator teacher in the process of finding its solutions is an important point in the implementation of the facilitative approach, as solving mathematical problems involves the use of different methods, but the solution is usually the only possible. In addition, the possession of fundamental mathematical knowledge on the chosen topic, necessary for practical application, is one of the factors in the successful implementation of the facilitative approach.

At the beginning of the lesson the teacher announces the topic and justifies its choice for further research and discussion. Students are divided into groups, each of which receives its own task to find the sum of the degree series. Tasks for groups, depending on the level of students, are determined by the teacher. Students with a lower level of learning opportunities receive intermediate-level tasks, and students with a higher level of learning opportunities have higher-level tasks. They investigate the properties of the proposed series during the time determined by the teacher and determine the method for finding its sum, check the results using the MAPLE package. Then the groups exchange the terms of the tasks without their work. This process continues until each group has completed all the tasks. Then the results are discussed with a possible discussion, if necessary. At each stage, pedagogical support and support of students by the teacher are important. He manages the process itself, but does not put pressure on students, intervenes in the discussion as needed. If students are unable to come up with an idea, the teacher directs their work or solves a similar problem on the board. In the end, students must choose the best solution for each problem.

When solving problems on this topic with the MAPLE package, keep in mind that the task of any sequence is identified with the function of a set on a set of natural numbers, i.e. each natural $n$ corresponds to some $f(n)$. Implementation of typical tasks does not usually require the connection of additional packages. To get a quick answer, it is sometimes necessary to limit the calculations to the actual area.

The following are solutions to typical problems of individual work on the topic.

*Problem* 1. Decompose into a Taylor series by powers $(x-1)$ a function $f(x)=\ln x$.

*Solutions*. To schedule functions in a row, the *series* command is provided, which allows you to find the answer to a certain order of accuracy.

**> f := ln(x);**

$$f := \ln(x)$$

**> series(f, x = 1, 5); F := convert(%, polynom);**

$$F := x - 1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$$

Let us check the result by constructing a function and its approximation in the form of a polynomial near the point $x = 1$ (Fig. 2).

**> plot([f, F], x = .1 .. 3, color = [red, blue]);**

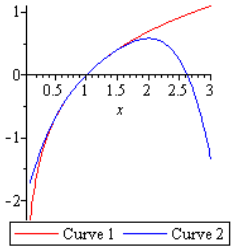


Figure 2. Illustration for the task

*Answer*:

$$x - 1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + O\left((x-1)^5\right)$$

*Problem* 2. Construct the integration domain $z = x^2 + y^2, z = 0, x = 1, y = 2, x = 0, y = 0$.

*Solutions*. For construction we will use not only commands of a package of graphics plots, but also commands of assignment of sequences.

Construction on one drawing of all surfaces that limit the body, gives only an idea of its contours. This task can be considered simple due to the simplified version of the sides of the body, it is limited by planes, which can be seen by turning the structure composed of given surfaces (Fig. 3).

**>with(plots):**

**>implicitplot3d([z = x^2+y^2, z = 0, x = 1, y = 2, x = 0, y = 0], x = 0 .. 1, y = 0 .. 2, z = 0 .. 5, color = [red, red, grey, green, grey, green])**

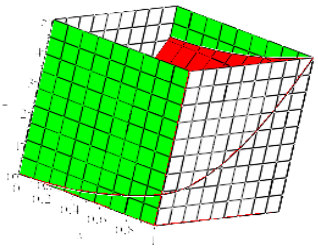

Figure 3. Illustration for the task

To obtain clear intersection curves of each of the pairs of surfaces, we use the intersectplot command and create geometric objects, which are, in particular, separate intersection curves of the paraboloid $z = x^2 + y^2$ with planes.

**> p1 := intersectplot(z = x^2+y^2, x = 1, x = 0 .. 1, y = 0 .. 2, z = 0 .. 5, color = black, thickness = 2);**

**> p2 := intersectplot(z = x^2+y^2, y = 2, x = 0 .. 1, y = 0 .. 2, z = 0 .. 5, color = black, thickness = 2);**

**> p3 := intersectplot(z = x^2+y^2, y = 0, x = 0 .. 1, y = 0 .. 2, z = 0 .. 5);**

**> p4 := intersectplot(z = x^2+y^2, x = 0, x = 0 .. 1, y = 0 .. 2, z = 0 .. 5);**

Separately we will create geometric objects that delimit the desired body from above and below.

**> p5 := implicitplot3d(z = x^2+y^2, x = 0 .. 1, y = 0 .. 2, z = 0 .. 5, color = green);**

**> p6 := implicitplot3d(z = 0, x = 0 .. 1, y = 0 .. 2, z = 0 .. 5, color = grey);**

We will now construct the sides of the body with the *polygonplot3d* commands, for which we will separately calculate the points (sequences) on the intersection curves of the paraboloid with the vertical planes, which will be connected by a closed polyline.

**> k1 := seq([1, (1/4)*T, 1+(1/16)*T^2], T = 0 .. 8);**

$$k1 := [1, 0, 1], \left[1, \frac{1}{4}, \frac{17}{16}\right], \left[1, \frac{1}{2}, \frac{5}{4}\right], \left[1, \frac{3}{4}, \frac{25}{16}\right], [1, 1, 2], \left[1, \frac{5}{4}, \frac{41}{16}\right], \left[1, \frac{3}{2}, \frac{13}{4}\right], \left[1, \frac{7}{4}, \frac{65}{16}\right], [1, 2, 5]$$

**> k2 := seq([(1/4)*T, 2, 4+(1/16)*T^2], T = 4 .. 0, -1);**

$$k2 := [1, 2, 5], \left[\frac{3}{4}, 2, \frac{73}{16}\right], \left[\frac{1}{2}, 2, \frac{17}{4}\right], \left[\frac{1}{4}, 2, \frac{65}{16}\right], [0, 2, 4]$$

**> k3 := seq([(1/4)*T, 0, (1/16)*T^2], T = 4 .. 0, -1);**

$$k3 := [1, 0, 1], \left[\frac{3}{4}, 0, \frac{9}{16}\right], \left[\frac{1}{2}, 0, \frac{1}{4}\right], \left[\frac{1}{4}, 0, \frac{1}{16}\right], [0, 0, 0]$$

**> k4 := seq([0, (1/4)*T, (1/16)*T^2], T = 8 .. 0, -1);**

$$k4 := [0, 2, 4], \left[0, \frac{7}{4}, \frac{49}{16}\right], \left[0, \frac{3}{2}, \frac{9}{4}\right], \left[0, \frac{5}{4}, \frac{25}{16}\right], [0, 1, 1], \left[0, \frac{3}{4}, \frac{9}{16}\right], \left[0, \frac{1}{2}, \frac{1}{4}\right], \left[0, \frac{1}{4}, \frac{1}{16}\right], [0, 0, 0]$$

**> p7 := polygonplot3d([[1, 2, 0], [1, 0, 0], k1, k2, [0, 2, 0]]);**

**> p8 := polygonplot3d([[0, 2, 0], k4]);**

**> p9 := polygonplot3d([[1, 0, 0], k3]);**

All geometric objects are displayed in one figure with the *display* command (Fig. 4).

**> display(p1, p2, p3, p4, p5, p6, p7, p8, p9, orientation = [-30, 60, 0]);**

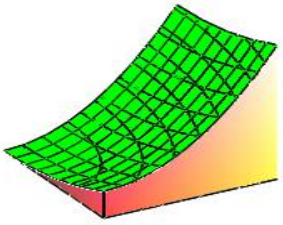

Figure 4. Illustration for the task

If students have not been able to generate any ideas, the teacher directs their work or suggests a solution to a similar problem on the board. As a result, students must choose the optimal method of solving each problem, which leads to the correct result.

The wider use of distance learning caused by the pandemic leads to the involvement of digital tools for synchronous and asynchronous interaction of participants in the educational process. The most popular in these conditions are the platform "MOODLE", electronic textbooks, social networks, cloud technologies (GOOGLE-disk), online boards, sites-designers of educational tasks and more. For lectures and seminars in mathematics, we provide programs "ZOOM" and "Teams", the capabilities of which allow you to transfer facilitative technologies to the online environment. The tools of these services allow you to group students and create closed channels in them (this can only be done by the teacher, students do not have the ability to edit groups and move from one group to another). The facilitator can be in touch with one active communication group at a time and with three others on standby. When completing a task, members of each group can chat, create an instant video conference and collaborate on a document [10].

### *E. The results of the pedagogical experiment*

We were also interested in how the use of a facilitative approach in the teaching of mathematical disciplines of future mathematics teachers with the support of the MAPLE package affect the effectiveness of the educational process. We conducted research on the basis of Makarenko Sumy State Pedagogical University, the study involved groups of students 2-4 courses future teachers of mathematics. The study was conducted for two

years in the first half of the school year, when students studied a course in mathematical analysis with the support of the MAPLE. We compared the performance of these student groups in the first half of the academic year with their performance in mathematics analysis, but in the previous semester, when training was conducted in the usual most common way in the lecture-practical system. The results are shown in table 2.

TABLE II. RESULTS OF EXPERIMENTAL TRAINING

| Group of students | Progress of students: success of training | |
|---|---|---|
| | ordinary learning | experimental training |
| *2 course (12 students)* | 48% | 54% |
| *3 course (10 students)* | 53% | 68% |
| *4 course (16 students)* | 56% | 72% |

As you can see, the use of a facilitative approach in teaching mathematics to future mathematics teachers with the support of MAPLE supports the desire of higher education students to self-development, self-realization, self-improvement, helps to reveal their abilities and cognitive abilities. An important point of successful use of facilitation technologies is the possession of fundamental mathematical knowledge on the chosen topic, necessary for practical application. Therefore, we believe that the experience of using a facilitative approach in the teaching of future mathematics teachers in the teaching of mathematical disciplines using the mathematical package MAPLE is positive and deserves to be disseminated.

## III. CONLUSION

Describing the practical possibilities of facilitative technologies confirms their effectiveness in creating favorable conditions for training future mathematics teachers in mathematics analysis classes using the MAPLE package, to increase the motivational component of educational activities, disclosure of creative potential and subjective involvement in working on problems and situations. The facilitative approach in higher education provides the choice and implementation of individual educational trajectories of students, promotes the development of their personal qualities.

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